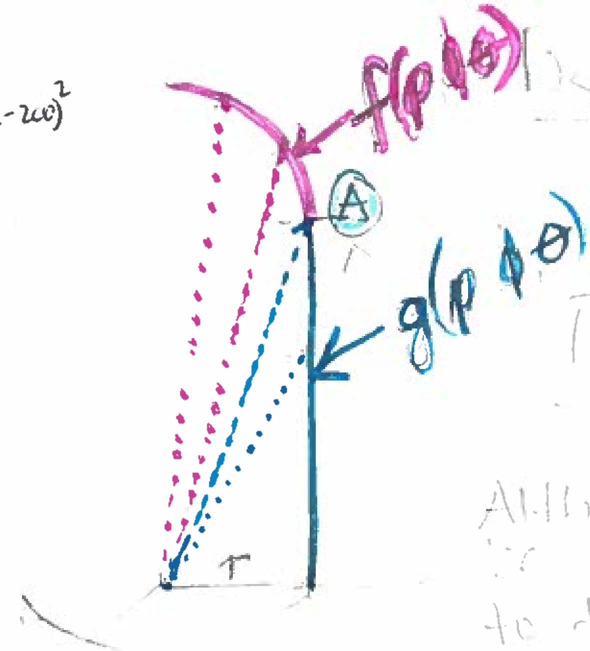
$50^2 = x^2+y^2+(z-200)^2$



[illegible]

[illegible] (r is constant)

$x^2+y^2=50^2$

Although [illegible] EASIER to do the cylinder in CYLINDRICAL coords.

$z=200$

$\rho\cos\phi$

$\phi=\cos^{-1}(\rho)$

ϕ goes from 0 to ϕ at point A for f()

ϕ goes from ϕ at point A to $\pi/2$ for g()

θ goes from 0 to 2π

QUESTION:

① How do you get [illegible] centroid / center of mass [illegible] on these [illegible] integrals?

Change of variables

Ex

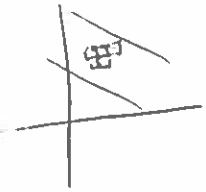
$$\iint_R \frac{e^{x-y}}{x+y}\,dx\,dy$$

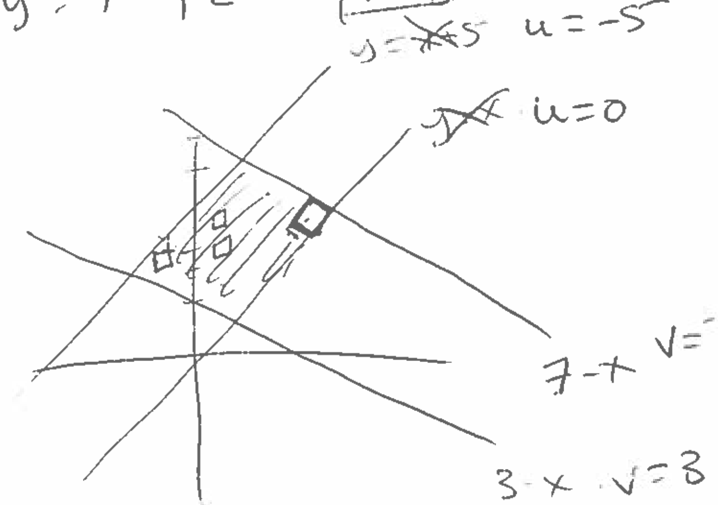
$y = x \leftarrow (x-y) = 0$

$y = x+5 \leftarrow (x-y) = -5$

$y = 3-x \leftarrow \boxed{x+y} = 3$

$y = 7-x \leftarrow \boxed{x+y} = 7$





$u = x - y$

$v = x + y$

$$\int_{u=-5}^{0}\int_{v=3}^{7} \frac{e^{u}}{v}\ \text{"}J\text{"}\,dv\,du$$

Jacobian

$\frac{1}{2}$

$= \frac{1}{2}\int_{-5}^{0} e^{u}\,du \cdot \int_{3}^{7} \frac{1}{v}\,dv$

$= \frac{1}{2}(e^{-5}-1)(\ln 7 - \ln$

$= \frac{1}{2}\ln\left(\frac{7}{3}\right)(e^{0} - e^{-5})$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$u = x - y$

$v = x + y$ → add together $2x = u + v$

→ subtract $2y = v - u$

$x = \frac{u+v}{2}$

$y = \frac{v-u}{2}$

$$\begin{vmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{vmatrix}$$

$= \frac{1}{4} - -\frac{1}{4} = \frac{1}{2}$

$\frac{\partial x}{\partial u} = \frac{1}{2}$ $\quad \frac{\partial x}{\partial v} = \frac{1}{2}$

$\frac{\partial y}{\partial u} = -\frac{1}{2}$ $\quad \frac{\partial y}{\partial v} = \frac{1}{2}$

$u = 9x + 9y + 3z$

$v = 9x + 18y$

$w = 9y + 3z$

$$\text{rref}\begin{array}{ccc|ccc} x & y & z & u & v & w \\ \cancel{u} & \cancel{v} & \cancel{w} & \cancel{x} & \cancel{y} & \cancel{z} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 9 & 9 & 3 & 1 & 0 & 0 \\ 9 & 18 & 0 & 0 & 1 & 0 \\ 0 & 9 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/9 & 0 & -1/9 \\ 0 & 1 & 0 & -1/18 & 1/18 & 1/18 \\ 0 & 0 & 1 & 1/6 & -1/6 & 1/6 \end{array}\right]$$

matrix = $\boxed{2^{nd}}$ $\boxed{x^{-1}}$

EDIT [A]

3×6

$$\left[\begin{array}{cccccc} 9 & 9 & 3 & 1 & 0 & 0 \\ 9 & 18 & 0 & 0 & 1 & 0 \\ 0 & 9 & 3 & 0 & 0 & 1 \end{array}\right]$$

QUIT

matrix > OPS

rref

matrix 1: [A]

rref([A])

$x = 1/9\,u - 1/9\,w$

$y = -1/18\,u + 1/18\,v + 1/18\,w$

$z = 1/6\,u - 1/6\,v + 1/6\,w$

$$A^{-1}$$

$$J = \det\begin{vmatrix} 1/9 & 0 & -1/9 \\ -1/18 & 1/18 & 1/18 \\ 1/6 & -1/6 & 1/6 \end{vmatrix} = .00205$$

$1/486$

$x = r\cos\theta$
$y = r\sin\theta$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta$$

$$= r$$

$$\langle x - 10xy ,\ y^2 - 5x^2 \rangle$$

$f_x = x - 10xy \qquad f_y = y^2 - 5x^2$

$= \frac{x^2}{2} - 5x^2y + \varphi_1(y) \qquad = \frac{y^3}{3} - 5x^2y + \varphi_2(x)$

$$f(x,y) = \frac{x^2}{2} - 5x^2y + \frac{y^3}{3} + C$$

f_{xy}

Work = Force x Distance.

$\vec{F}(x,y)$

$W = \int_C \vec{F} \cdot d\vec{r}$

$\vec{r}$ = parameterized curve

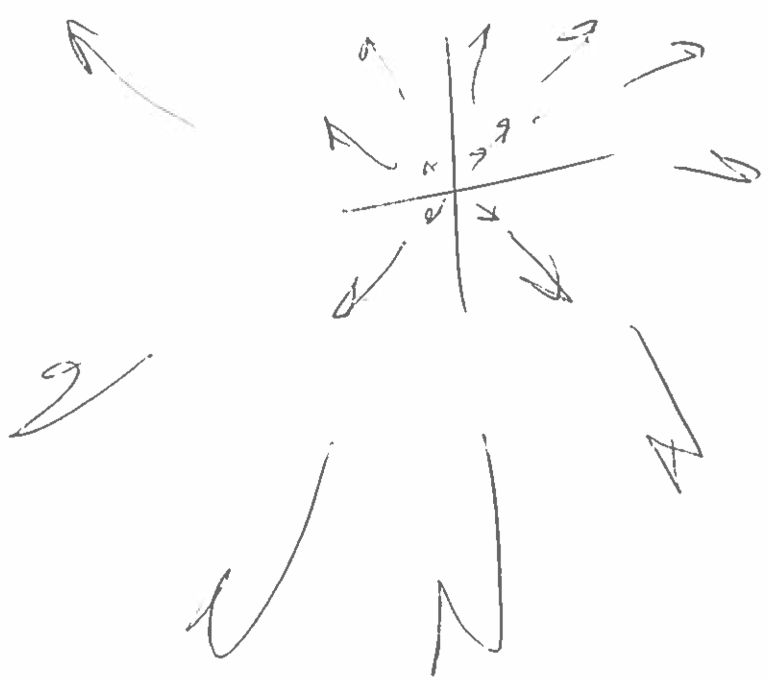
$(3,1) \rightarrow (7,4)$

$\vec{F} = \langle 6x, 6y \rangle$

$\langle 4, 3 \rangle$

$\vec{r} = \langle 3+4t, 1+3t \rangle$

$d\vec{r} = \langle 4, 3 \rangle dt$



$\int_0^1 \langle 6(3+4t), 6(1+3t) \rangle \cdot \langle 4, 3 \rangle dt$

$\int$

conservative $\vec{F} = \langle y, 0, z \rangle$

$f(x,y,z) = xy + \frac{z^2}{2} + c$

$f_x = y \quad f_y = 0 \quad f_z = z$

$xy \qquad \frac{z^2}{2}$

Not conservative