

# Lagrange Multiplier

$$\nabla f = \lambda \nabla g$$

~~$\nabla f =$~~

~~$\nabla f$~~

$\underline{V}$   $V = xyz$

$$x^2 + y^2 + z^2 = 27$$
$$xy + yz + xz = 27$$

$$\nabla f = \lambda \nabla g =$$

$$\langle yz, xz, xy \rangle = \lambda \langle y+z, x+z, y+x \rangle$$

$$x \cdot yz = \lambda (y+z) \cdot x$$

$$y \cdot xz = \lambda (x+z) \cdot y$$

$$z \cdot xy = \lambda (y+x) \cdot z$$

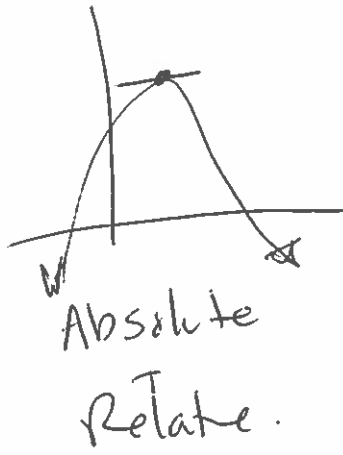
$$x \lambda (y+z) = y \lambda (x+z) = z \lambda (y+x)$$

$$xy\lambda + xz\lambda = xy\lambda + zy\lambda$$

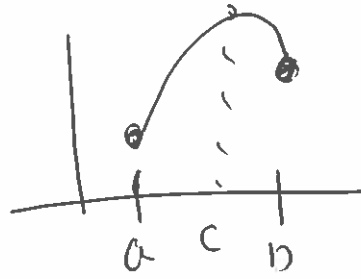
$$x = y$$

$$y = z$$

# MAX / MINS



$$f'(x) = 0$$



$$f'(x) = 0$$

$$f(a)$$

$$f(b)$$

$$f(c) - \text{MAX}$$

$$f(x, y) =$$

$$A = x \cdot y$$

$$A(x, y) = xy$$

Constraint.

$$\text{Perimeter} = 1000'$$

$$2x + 2y = 1000$$

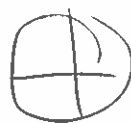
$$y = \frac{1000 - 2x}{2}$$

$$A(x, y) = x \left( \frac{1000 - 2x}{2} \right)$$

$$f_x = 0, f_y = 0 \Rightarrow \text{critical points}$$

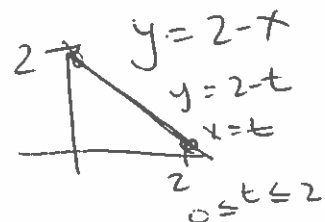
Finding MAX/mins  
over Region  $R$

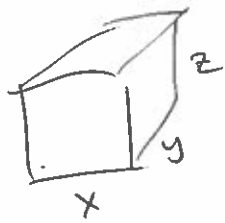
Parametrize the  
boundary for  $R$



$$x = \cos t$$

$$y = \sin t$$





$$\text{Surface Area} = 54$$

$$2xy + 2yz + 2xz = 54$$

$$z(2y + 2x) = 54 - 2xy$$

$$z = \frac{54 - 2xy}{2y + 2x}$$

$$V(x, y, z) = V = xyz$$

$$V(x, y) = xy \left( \frac{54 - 2xy}{2y + 2x} \right) = \frac{54xy - 2x^2y^2}{2y + 2x}$$

$$V_x = \frac{(2y + 2x)(54y - 4xy^2)}{(2y + 2x)^2} = 0$$

$$\cancel{y = x}$$
$$\cancel{y = 0}$$

$$V_y =$$

$$54 - 4xy = 0$$

$$xy = \frac{54}{4}$$

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<p>GROUP NAME: <u>B.0</u></p> <p>Logo:</p>	<p>Student Names (First and Last)</p> <p>Speaker/Presenter: <u>Shawnee</u></p>
<p>Date: _____</p> <p>Topics:</p>	<p>Writer/Prep: <u>Piyush</u></p> <p>QC/Leader: <u>Scenes</u></p>

Instructions:

$$f = xyz$$

$$x + y + z = 34$$

$$x + y - z = 0$$

$$\nabla f(x) = \lambda \nabla g_1(x) + \mu \nabla g_2(x)$$

5 equations

- 1.  $yz = \lambda + 2\mu$
- 2.  $xz = \lambda + \mu$
- 3.  $xy = \lambda - \mu$
- 4.  $x + y + z = 34$
- 5.  $x + y - z = 0$

$$\text{Subtract eq 4 - eq 5}$$

$$2z = -34$$

$$z = -17$$

know  $z = -17$

$$x + y + (-17) = 34$$

$$x + y = 51$$

$$2x = 51$$

$$x = \frac{51}{2} = 25.5$$

know  $z = -17$

$$x + y - (-17) = 0$$

$$x + y + 17 = 0 \implies x + y = -17$$

GROUP NAME: <u>42</u>	Student Names (First and Last)
Logo:	Speaker/Presenter: _____
Date: _____	Writer/Prep: <u>Ben &amp; Kyle</u>
Topics:	QC/Leader: <u>Gary</u>

Instructions:  $f(x, y, z) = x^2 + y^2 + z^2$   $x + 2y + 4z = -15$   
 $y + z = 0$

$$\nabla f = \lambda \nabla g + \mu \nabla h$$

$$\langle 2x, 2y, 2z \rangle = \lambda \langle 1, 2, 4 \rangle + \mu \langle 0, 1, 1 \rangle = \langle -10, -20, -40 \rangle + \langle 0, 30, 30 \rangle$$

$$= \langle -10, 10, -10 \rangle$$

$$\langle x, y, z \rangle = \langle -5, 5, -5 \rangle$$

$$2x = \lambda$$

$$2y = 2\lambda + \mu$$

$$+ 2z = 4\lambda + \mu$$

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$$2y + 2z = 6\lambda + 2\mu$$

$$0 = 3\lambda + \mu$$

$$x = -5$$

$$y = 5$$

$$z = -5$$

$$x + 2y + 4z + 15 = 0$$

$$x + 2y - 4y + 15 = 0$$

$$x - 2y + 15 = 0$$

$$x - (2\lambda + \mu) + 15 = 0$$

$$2x - 4\lambda - 2\mu + 30 = 0$$

$$\lambda - 4\lambda - 2\mu + 30 = 0$$

$$-3\lambda - 2\mu + 30 = 0$$

$$3\lambda + \mu + \mu = 30$$

$$0 + \mu = 30$$


$$\mu = 30$$

$$0 = 3\lambda + \mu$$

$$0 = 3\lambda + 30$$

$$\lambda = -10$$

MAX: 75 MIN

<p>GROUP NAME: <u>Team OP</u></p> <p>Logo: </p>	<p>Student Names (First and Last)</p> <p>Speaker/Presenter: <u>Amanda Fowler</u></p>
<p>Date: <u>10/10/13</u></p> <p>Topics:</p>	<p>Writer/Prep: _____</p> <p>QC/Leader: <u>Javier Blanco</u></p>

Instructions:

$f(x, y)$     constraint  $(g(x, y) \leq c)$

$f(x, y) = 10x^2 - y$     subject to  $x^2 + y^2 \leq 3$     ← constraint

$\nabla f = \lambda \nabla g$

$x^2 + y^2 - 3 = 0$

$$\left[ \begin{array}{l} \nabla f = \langle 20xy, 10x^2 \rangle \\ \nabla g = \langle 2x, 2y \rangle \\ \textcircled{x^2 + y^2 - 3 = 0} \end{array} \right]$$

$\lambda = 0$

$\langle 20xy, 10x^2 \rangle = 0$

$\langle 0, y \rangle$  ~~max~~

$\langle 20xy, 10x^2 \rangle = \lambda \langle 2x, 2y \rangle$

$20xy = \lambda 2x$      $\frac{20xy}{2x} = \lambda$      $10y = \lambda$

$10x^2 = \lambda 2y$      $10x^2 = 10y 2y$      $\frac{10x^2}{10} = \frac{20y^2}{10}$

$x^2 = 2y^2$

$x = y\sqrt{2}$

$\left[ \begin{array}{l} x^2 + y^2 - 3 = 0 \\ \textcircled{x^2 + y^2 - 3 = 0} \end{array} \right]$

$(y\sqrt{2})^2 + y^2 - 3 = 0$

$2y^2 + y^2 - 3 = 0$

$3y^2 - 3 = 0$

$y^2 = 1$

$10 = \lambda$

$x = \pm\sqrt{2}$

$y = \pm 1$

$(\pm\sqrt{2}, \pm 1)$

MAX 20  
MIN -20

<b>GROUP NAME:</b>	<b>Student Names (First and Last)</b>
<b>Logo:</b>	<b>Speaker/Presenter:</b> _____
<b>Date:</b> _____	<b>Writer/Prep:</b> _____
<b>Topics:</b>	<b>QC/Leader:</b> _____

**Instructions:**

*[Faint handwritten notes and diagrams are visible in the background.]*

$(2, 6)$   $(2, 6)$   
 $(-2, -6)$   $(-2, 6)$   
(

MAX: 72  
MIN: -72

*[Two blue circles are drawn at the bottom of the page.]*