Undecided Kate Moran Jose Pierre
Orsana P.

2. Set up the system of equations used to find the equation of the parabola $y = Ax^2+Bx+C$ going thru the points (1,2),(2,4),(3,2),(4,0).

a) What would the system of equations look like in reduced row echelon form?

$A+B+C=2$
$4A+2B+C=4$
$9A+3B+C=2$
$16A+4B+C=0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 2 \\ 16 & 4 & 1 & 0 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$A=0$
$B=0$
$C=0$
$0A+0B+0C=1$

b) How many solutions are there?

Inconsistent

c) Find the parabola that is closest to the all four points using matrices.

$$\vec{Z} = (A^TA)^{-1}A^T\vec{b}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4 & 9 & 16 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 0 \end{bmatrix}$$

$$\vec{Z} = (A^TA)^{-1}A^T\vec{b} \overset{\text{(calculator)}}{=} \begin{bmatrix} -1 \\ \frac{21}{5} \\ -1 \end{bmatrix}$$

$$\boxed{y = -x^2 + \frac{21}{5}x - 1}$$

Math busters

Sneha Rangu
Varun Jain
Jason DeLeon

3. $C = \begin{bmatrix} 1 & 2 & 0 & 7 \\ 1 & 1 & 2 & 5 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

a. Find det C → $\det C = 1(1(21)) - 1(2(21))$
$\boxed{\det C = -21}$ $= 21 - 42 = -21$

b. What is the relationship between the determinant and the inverse

$C^{-1} = \frac{adj(C)}{\det(C)}$ adj = adjugate

If the determinant is zero, then the matrix has no inverse

c. Find C^{-1} ⇒ In calculator → 2nd matrix → edit [C] → [C]$^{-1}$ → enter

$$C^{-1} = \begin{bmatrix} -1 & 2 & -4/3 & -3/7 \\ 1 & -1 & 2/3 & -2/7 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/7 \end{bmatrix}$$

(4) geometric interpretation for $\omega = \{x \cdot x \begin{bmatrix} x \\ y \\ z \end{bmatrix},$

$x = 0,\ y + z = 0\}$

a) it is a ~~plane~~ Line that crosses the origin

b) to show it is a subspace you need

- the zero vector in subspace $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ✓
- $v_1 + v_2$ is in the ~~space~~ if V_1, V_2 in subsp
- multiples of v_1 are included

c) determine if the above subset is a subspace

yes ? $[0,0,0]$

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} x = 0 \\ y + z = 0 \end{matrix}$ so $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ in subsp

$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \in$ space $\begin{matrix} x_1 = 0 \\ y_1 + z_1 = 0 \end{matrix}$ $\begin{matrix} x_2 = 0 \\ y_2 + z_2 = 0 \end{matrix}$

$\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix} = \begin{cases} x = 0 \\ y_1 + y_2 \end{cases}$ $\begin{cases} x_1 + x_2 = 0 \\ + z_1 + z_2 = 0 \end{cases}$

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Math Group Bishoi Anna Gregory Sam

5) $C = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{rref[A]} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1 + 2x_2 = 0 \Rightarrow x_1 = -2x_2$

$2x_3 = 0$

$3x_4 = 0$

Null space: $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

a) pivot columns are 1, 4 and 3

b) free variables is 2

c) Rank = 3

<<HEFF>>

MARIAM

ROB

DOM

6) $C = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

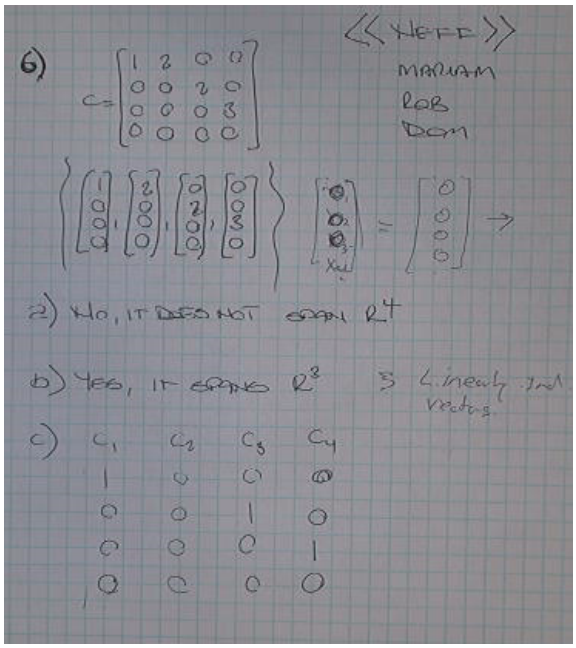
$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} \right\}$

a) No, IT DOES NOT SPAN R^4

b) YES, IT SPANS R^3

c)

C_1	C_2	C_3	C_4
1	0	0	0
0	0	1	0
0	0	0	1
0	0	0	0



6)

$$C = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

<<NEFF>>
MARIAM
ROB
DOM

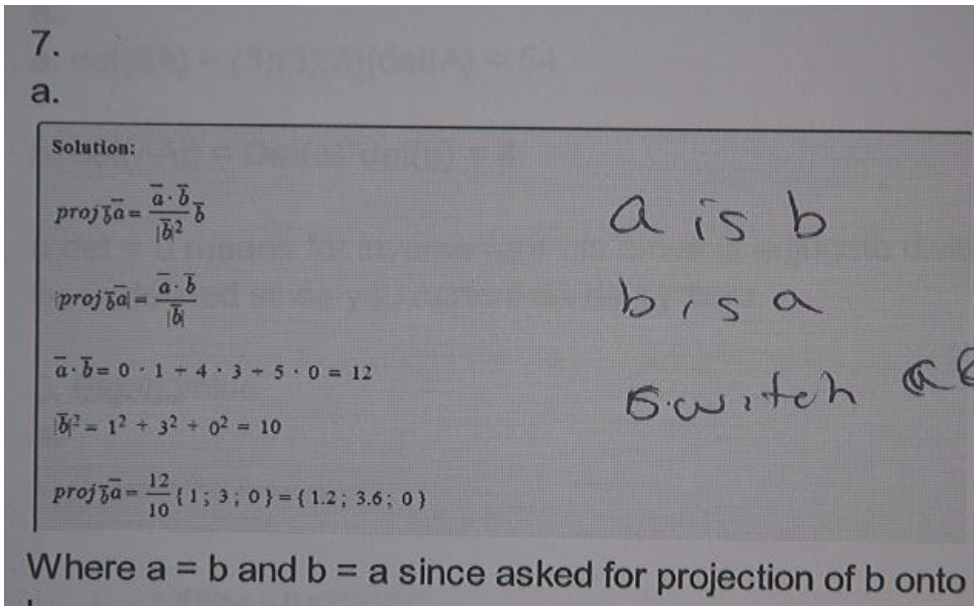
$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} \right\} \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$$

a) NO, IT DOES NOT SPAN R^4

b) YES, IT SPANS R^3 3 Linearly Ind vectors

c)

C_1	C_2	C_3	C_4
1	0	0	0
0	0	1	0
0	0	0	1
0	0	0	0



7.

a.

Solution:

$$proj_{\bar{b}}\bar{a} = \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|^2}\bar{b}$$

$$|proj_{\bar{b}}\bar{a}| = \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$$

$$\bar{a}\cdot\bar{b} = 0\cdot 1 + 4\cdot 3 + 5\cdot 0 = 12$$

$$|\bar{b}|^2 = 1^2 + 3^2 + 0^2 = 10$$

$$proj_{\bar{b}}\bar{a} = \frac{12}{10}\{1;3;0\} = \{1.2;3.6;0\}$$

a is b

b is a

Switch a

Where a = b and b = a since asked for projection of b onto

b.

Solution:

$$proj_{\bar{b}}\bar{a} = \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|^2}\bar{b}$$

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+

Solution:

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$$|proj_{\bar{b}}\bar{a}| = \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$$

$$\bar{a}\cdot\bar{b} = 0\cdot 0 + 4\cdot 2 + 5\cdot 7 = 43$$

$$|\bar{b}|^2 = 0^2 + 2^2 + 7^2 = 53$$

$$proj_{\bar{b}}\bar{a} = \frac{43}{53}\{0;2;7\} = \{0;\frac{86}{53};\frac{301}{53}\}$$

Projecting onto a space is just sum of projections individual

8.

a. det(3A) = (3)(3)(3)(det(A) = 54

b. det(AA^T) = Det(a)*det(a) = 4

2 2

c.det = 0 means for inverse formula since is adjugate divided by determinant, the inverse would be undefined since you cannot divide by zero.

0

$$\det\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{vmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 54$$

Eigenvalue problem

3x3 or 2x2 with real eigenvalues, integers

5.2 Determine the given subset of C[-1,1] is also a vector space.

$$F = \left\{ f(x) : \int_{-1}^{1} f(x)dx = 0 \right\}$$

contains f(x)=0

cf(x) in too

f + g = 0

$$R = \left\{ \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix} \right\}$$

1	0	...
2	1	
0	3	
0	2	