Please show ALL your own work for full credit- even when using a calculator

1. LINEAR EQUATIONS AND MATRICES (2 weeks) = chapter 1 & 2 & 4
At the end of Unit 1, the student should be able to:
✓ calculate the dot product between vectors. (Course Goal 1)
✓ calculate the length of a vector. (Course Goal 1, Gen Ed Goal 2)
✓ explain the concept of orthogonality of vectors. (Course Goal 1)
✓ create a unit vector from a given vector. (Course Goal 1)
✓ demonstrate that one vector is a linear combination of given vectors. (Course Goal 1)
✓ apply matrix operations. (Course Goal 1)
✓ set up matrix operations using proper technology. (Course Goal 1,12)

2. DETERMINANTS AND INVERSES (2 weeks) Chpt 2 & 3
At the end of Unit 3, the student should be able to:
✓ calculate the inverse of a matrix. (Course Goal 2, 12)
✓ solve systems by using row reduction and LU factorization. (Course Goal 2, 12)
✓ recognize the connection between the elimination process and factoring a matrix. (Course Goal 2)
✓ define determinant. (Course Goal 10)
✓ calculate the determinant of a square matrix and interpret it in terms of invertibility of a matrix. (Course Goal 4)
□ apply the basic properties of determinants. (Course Goal 4, 12)
✓ apply Cramer's Rule to solve systems of equations and volume problems. (Course Goal 4, 12)
chpter 2.4 to 3.2

3. VECTOR SPACES AND SUBSPACES (3.5 weeks) = chpt 5 & chpt 6
At the end of Unit 3, the student should be able to:
✓ explain the defining properties of a vector space. (Course Goal 3)
✓ construct examples of vector spaces. (Course Goal 3)
✓ explain why a set defined with the necessary operations is or is not a vector space. (Course Goal 3)
✓ explain why a subset of a given vector space is or is not a subspace. (Course Goal 3)
✓ define the span of a set of vectors. (Course Goal 3)
✓ determine if a collection of vectors from a given vector space is a spanning set for the vector space. (Course Goal 3, 12)
✓ define linear independence. (Course Goal 3 and 5)
✓ calculate whether or not a given set of vectors is linearly independent. (Course Goal 5,12)
□ calculate the rank of a given matrix. (Course Goal 5, 12)
□ explain the defining properties of a basis for a vector space. (Course Goal 5)
□ determine if a given set of vectors from a vector space is or is not a basis for the space. (Course Goals 5, 12)
□ explain what is meant by the dimension of a vector space. (Course Goal 5)MAT208 Course Outline Fall 2012
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□ state the properties of subspaces and the relationships among the four fundamental subspaces of a matrix. (Course Goals 3 and 5, Gen Ed Goals 1 and 2)
□ explain why the equation Ax=b is consistent if and only if b is in the column space of A. (Course Goals 3 and 5, Gen Ed Goals 1 and 2)
□ discuss how linear independence, spanning sets, basis and dimension are related. (Course Goals 5 and 12)
5.1 to 6.4

$$\vec{V} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \sqrt{\vec{V}^T \cdot \vec{V}} =$$ 1

2×1

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1^2 + 2^2$$

$$\| \vec{V} \| = \sqrt{V^T \cdot V}$$

Orthogonality $\quad V \cdot W = 0$

$V \perp W$

Unit vector $= \dfrac{\vec{V}}{\| \vec{V} \|}$

$\vec{V}, \vec{W} \qquad \vec{P}$

$$a\vec{V} + b\vec{W} = \vec{P}$$

$$\text{rref} \left[\vec{V} \;\vdots\; \vec{W} \right] = \left[\vec{b} \right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array} \right]$$

$$\vec{A}^{-1}\vec{A}\vec{x} = \vec{A}\vec{b}$$

$$\vec{x} = \vec{A}^{-1}\vec{b}$$

$$2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

INVERSE

$$\left[\begin{array}{c|c} A & I \end{array}\right]$$

$$\left[\begin{array}{c|c} I & A^{-1} \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$$

$R2 - 3R1$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right]$$

$R\cancel{2}1 + R2$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array}\right]$$

$R2 / -2$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right]$$

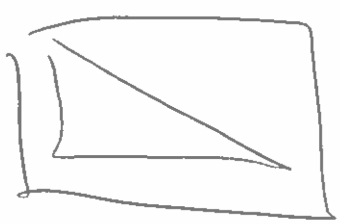
Check

$$\underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \underset{A^{-1}}{\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}} = \underset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

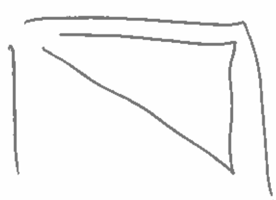
or calculate A^{-1}.

$$\underset{E_1}{\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

Lower



Upper



$$\underset{\text{upp- } E_2}{\begin{bmatrix} 1 & \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\underset{E_3}{\begin{bmatrix}1 & 0\\ 0 & -\tfrac{1}{2}\end{bmatrix}}\begin{bmatrix}1 & 0\\ 0 & -2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$ 4

$$E_3 E_2 E_1 A = I$$

$$A = \underbrace{E_3^{-1} E_2^{-1} E_1^{-1}}_{\text{Factors.}} I$$

$$\underset{E_2}{\begin{bmatrix}1 & \\ 0 & \end{bmatrix}}\ \underset{E_2^{-1}}{\begin{bmatrix}1 & \\ & 1\end{bmatrix}} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Determinant - Number from a Square matrix that if Zero means matrix has no inverse.

C

Cramers Rule.

$$\det \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 5 & 6 \\ 0 & \cancel{2} & 7 & 1 \\ 0 & 0 & 0 & \cancel{8} \end{bmatrix} = 21$$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ & & & & \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 2 \\ 0 & 2 & 3 & 6 & 3 \\ 0 & 0 & 7 & 1 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$$x_4 = \frac{\det \begin{bmatrix} \cancel{4} & 2 & 3 & \cancel{4}\,2 \\ 0 & 2 & 3 & \cancel{6}\,3 \\ 0 & 0 & 7 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}}{21} = \frac{21 \cdot 5}{21} = 5$$

Vectors Space

① $\vec{0}$, ② $\vec{x}$ $\Rightarrow$ $a\vec{x}$

③ $\vec{x}, \vec{y}$ $\Rightarrow$ $\vec{x} + \vec{y}$

Vector Space (6)

$\vec{v} \cancel{\in} \{\vec{v} : x^2 = y\}$ $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix} \begin{bmatrix}0\\0\end{bmatrix}$

NOT

$7\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}7\\7\end{bmatrix}$ Not

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$\{\vec{v} : y = x, z = 0\}$

$\begin{bmatrix}0\\0\\0\end{bmatrix}$ ✓ $\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}a\\a\\0\end{bmatrix}$ ✓

$\begin{bmatrix}a\\a\\0\end{bmatrix} \begin{bmatrix}b\\b\\0\end{bmatrix} \begin{bmatrix}a+b\\a+b\\0\end{bmatrix}$ ✓

<u>Subspace</u> = <u>Subset</u>

$$\vec{v} = \left\{ a\begin{bmatrix}1\\1\\0\end{bmatrix} + b\begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$$

$$a\begin{bmatrix}1\\1\\2\end{bmatrix} + \begin{bmatrix}2\\2\\4\end{bmatrix}$$

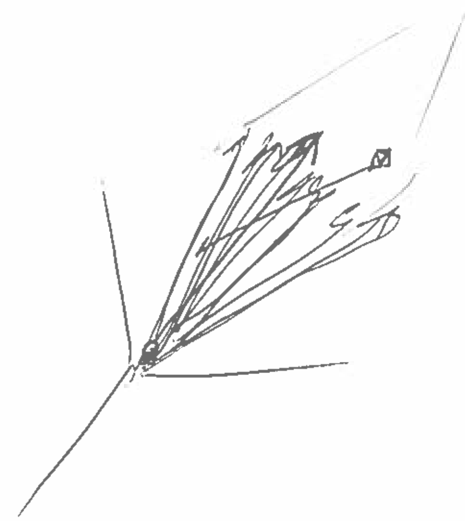
$\{$ x, y z = 1

$$\left\{ \begin{bmatrix}1\\1\\1\end{bmatrix} \begin{bmatrix}2\\2\\1\end{bmatrix} \begin{bmatrix}3\\3\\1\end{bmatrix} \begin{bmatrix}4\\4\\1\end{bmatrix} \right\}$$

Not A Subspace

Subset $\subset$ Vector space

Subspace also a <u>vector</u> <u>space</u>



$= 0$

<u>Span</u>

Image of $\vec{v}, \vec{w}$

$A\vec{x} = \vec{y}$

$$[\vec{v} \mid \vec{w}]\vec{x} = \underbrace{a\vec{v} + b\vec{w}}$$

$$\begin{bmatrix}a\\b\end{bmatrix}$$

$$\left\{ V : \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Basis

Vecter space in 3D
x - y plane

$$\left\{ V : \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cancel{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

span x, y plae

Not Linearly Independe

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$ spans line $y = x$

Linear Independance

$$\vec{V}_1, \vec{V}_2, \vec{V}_3 \ldots$$

$$a \vec{V}_1 + b \vec{V}_2 + c \vec{V}_3 = 0$$

$$\begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Linearly Indep

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & \cancel{0} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad 9$$

$b = - - e$

$$A \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Null space of A

Rank of A Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$ Linearly Ind

Span R^3

$$\text{rref} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 11 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \\ 0 \end{bmatrix}$$

Rank of 2

$$\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{\substack{\text{basis} \\ \text{Span } R^2}}$$

Null space

Rank of 1

$$\underbrace{a \begin{bmatrix} 6 \\ 6 \\ 4 \end{bmatrix}}_{\substack{\text{basis} \\ \text{Null space}}}$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

3×5 $\quad$ 5×1 $\quad$ 3×1

<u>Rank 2</u>

$a_1 + 2a_2 + 3a_4 = 0$

$a_3 + 4a_4 + a_5 = 0$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Span basis for Image.

Image combination of column space

$a_1 = -2a_2 - 3a_4$

$a_2 = a_2$

$a_3 = -4a_4 - a_5$

$a_4 = a_4$

$a_5 = a_5$

$$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Basis for Nullspace