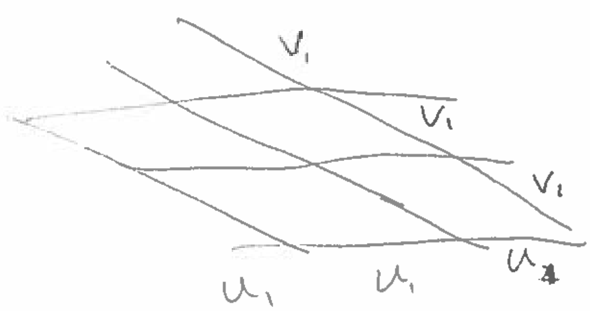
$U = \{ a\vec{V} \}$ a is any constant.

1. zero
2. b a $\vec{V}$
3. $aV_1 + bV_2 = (a+b)V$

$U = \{ aV_1 + bV_2 \}$

V_1, V_2 $\left\{ \begin{array}{l} V_1 = aV_2 \\ \text{Not plane -} \\ \text{just line} \end{array} \right.$



$\therefore V_1$ is multiple of V_2

Are linearly independant.

$$aV_1 + bV_2 = 0 \Rightarrow a = 0 = b$$

$$aV_1 + b(c)V_1 = 0$$

$$a + bc = 0$$

$$a = -bc$$ Not Linear. Ind.

$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ Span R^2

$$\begin{bmatrix} a \\ b \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$ Also Span R^2

$$\begin{bmatrix} 9 \\ 3 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 9 \\ 1 & -2 & 3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & -3 & 2 \end{array}\right]$$

$$\left\{ \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix} \text{ in } \mathbb{R}^4 \right\}$$

$$a\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + b\begin{bmatrix}1\\1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$ Then $a=b=0$

row 2: $b=0$

row 3: $a=0$

$a+b=0$

$$\begin{bmatrix}1\\2\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$ doesn't span $\mathbb{R}^4$

$$\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$$

Spanning = image

bases = smallest set a vectors to span some space

dimension = # vectors in bases

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Column space. $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix} \right\}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \#_1 & 0 \\ 0 & 1 & \#_2 & 0 \end{array}\right] \qquad \begin{array}{l} a = \#_1 \\ b = \#_2 \end{array}$$

Row spce $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}^T \right\}$

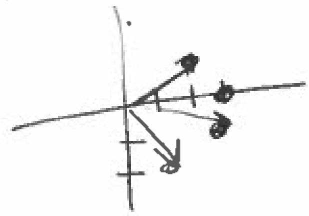
Orthogonal (means $\perp$)

$$\vec{A} \cdot \vec{B} = \|A\| \, \|B\| \cos\theta \qquad \cos(90^\circ) = 0$$

~~Exa 2/6~~

$$\left\{ \underset{\vec{i}}{\begin{bmatrix}1\\0\end{bmatrix}} \underset{\vec{j}}{\begin{bmatrix}0\\1\end{bmatrix}} \right\} \qquad \left\{ \underset{\vec{v}_1}{\begin{bmatrix}2\\1\end{bmatrix}}, \underset{\vec{v}_2}{\begin{bmatrix}1\\-2\end{bmatrix}} \right\}$$

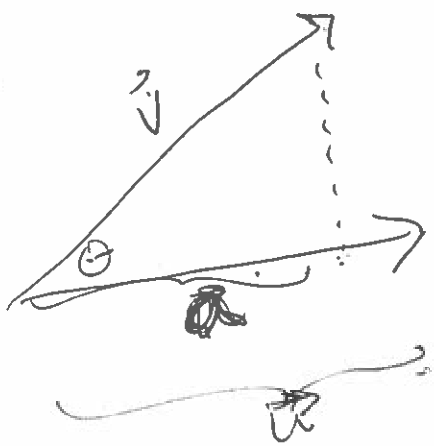
$2\vec{i} + \vec{j}$



$(3, -1)$

$$a\begin{bmatrix}2\\1\end{bmatrix} + b\begin{bmatrix}1\\-2\end{bmatrix} = \begin{bmatrix}3\\-1\end{bmatrix}$$

Projection

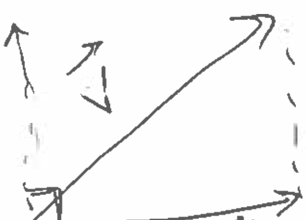


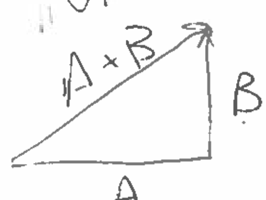
$$\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \cdot \vec{u}$$

$$\frac{\|u\|\|v\| \cos\theta}{\|u\|\|u\|}$$

$$\|a\| = \frac{\|u\|\|v\|}{\|u\|} \; \frac{\|a\|}{\|v\|}$$

$\|v\|$





$\vec{u}_1$ = keep

$\vec{u}_2 = \vec{v} - proj_{\vec{u}} \vec{v}$

$\vec{u}_1$ orthogonal to $\vec{u}_2$

orthoganalization → basis into perpendicular vectors

normalization → making the length = 1

orthonormal → unit vectors that are "⊥" to each othe

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$$

~~must~~ Linearly independant
Span $\mathbb{R}^3$
For a bases

$$\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}2\\1\\1\end{bmatrix} \quad \underset{v_1}{\begin{bmatrix}1\\1\\0\end{bmatrix}} \quad \underset{v_2}{\begin{bmatrix}1\\0\\1\end{bmatrix}}$$

bases of subspace

$$\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$\|v\| = \sqrt{2}$$

$$u_1 = \begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} - \operatorname{Proj}_{u_1} v_2$$

$$\frac{v_1 \cdot v_2}{\|v_1\|^2} v_1$$

$$\frac{1}{2} \cdot \begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} - \begin{bmatrix}1/2\\1/2\\0\end{bmatrix} = \begin{bmatrix}\cancel{\frac{\sqrt{2}-1}{\sqrt{2}}}\\-1/2\\1\end{bmatrix}$$

$$\begin{bmatrix}1/2\\-1/2\\1\end{bmatrix}$$

$$\begin{bmatrix}2\\1\\1\end{bmatrix} = \frac{3}{2}\begin{bmatrix}1\\1\\0\end{bmatrix}, \; 1\begin{bmatrix}1/2\\-1/2\\1\end{bmatrix}$$

$$\sqrt{\|1/4 + 1/4 + 1\|}$$

$$\sqrt{6/4} \quad \sqrt{3/2}$$

Orthonormal?

$$\begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\\0\end{bmatrix}, \begin{bmatrix}\frac{1}{2}\sqrt{3/2}\\-\frac{1}{2}\sqrt{3/2}\\1/\sqrt{3/2}\end{bmatrix}$$

$$V_1, V_2, V_3$$

$u_1 = V_1$ keep.

$$u_2 = V_2 - \mathrm{Proj}_{V_1} V_2$$

$$u_3 = V_3 - \mathrm{Proj}_{V_1} V_3 - \mathrm{Proj}_{V_2} V_3$$

Gram-Schmitt orthogonalization

5.61

$y = a_1 x + a_2$

(x_1, y_1)

(x_2, y_2)

(x_3, y_3)

$(1, 2)$

$(3, 4)$

$(5, 6)$

$2 = a_1 + a_2$

$4 = 3a_1 + a_2$

$6 = 5a_1 + a_2$

$$\begin{array}{cc} A & B \\ \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 5 & 1 \end{bmatrix} & \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \end{array}$$

$$Ax = b$$

$$(A^T A) z = A^T b$$

$$z = (A^T A)^{-1} A^T b$$

$$\left(\begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 5 & 1 \end{bmatrix} \right)^{-1}$$

$$\begin{bmatrix} 35 & 9 \\ 9 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

2×2 2×3 3×1

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y = a_1 + a_2 x + a_3 x^2$$

(0, 5)
(1, 2)
(2, 4)
(3, 7)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 2 \\ 4 \\ 7 \end{bmatrix}$$

$$\left(\overset{A'}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 9 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}} \right)^{-1} \left(\quad \right)$$

$$\left(\begin{bmatrix} 1 & 12 & 14 \\ 6 & 14 & 36 \\ 14 & 36 & 98 \end{bmatrix} \right)^{-1} \left(\quad \right) \left(\quad \right)$$

3×3 3×4 4×1