Fundamental Theorem of Cal.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

- Definite Integral
- Area under curve

$$F(x) = \int f(x)\,dx$$

U-Substitution

$$\int x \sin(x^2)\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int \sin(u)\,\frac{du}{2}$$

$$\frac{1}{2}\int \sin(u)\,du$$

$$-\frac{1}{2}\cos(u) + C$$

ANS $-\frac{1}{2}\cos(x^2) + C$

FT of C w/ u-sub.

Ex/ $$\int_3^5 \frac{\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int_{\ln 3}^{\ln 5} u\,du = \left.\frac{u^2}{2}\right|_{\ln 3}^{\ln 5}$$

$u(3) = \ln 3$

$$= \frac{(\ln 5)^2}{2} - \frac{(\ln 3)^2}{2}$$

Area under curve $\frac{\ln x}{x}$ from $x=3$ to $x=5$

$$\int_{x=a}^{b} f(x)\,dx \rightarrow \int_{u(a)}^{u(b)} f(u)\,du = F(u(b)) - F(u(a))$$

$$= F(u)\Big|_{u(a)}^{u(b)}$$

Ex $\int_{0}^{1} x\sqrt{x+5}\,dx \;=\; 1.18$

$u = x+5$
$du = dx$
$\hookrightarrow x = u-5$

$$\int_{u(0)=5}^{u(1)=6} (u-5)u^{1/2}\,du$$

$$\int_{5}^{6} u^{3/2} - 5u^{1/2}\,du$$

$$\frac{u^{5/2}}{5/2} - \frac{5u^{3/2}}{3/2}\Bigg|_{5}^{6}$$

$$\frac{2}{5}(6)^{5/2} - \frac{10}{3}(6)^{3/2}$$

$$-\left(\frac{2}{5}(5)^{5/2} - \frac{10}{3}(5)^{3/2}\right)$$

$$\frac{2}{5}(6)^{5/2} - \frac{10}{3}(6)^{3/2} - \frac{2}{5}(5)^{5/2} + \frac{10}{3}(5)^{3/2} = 1.18$$

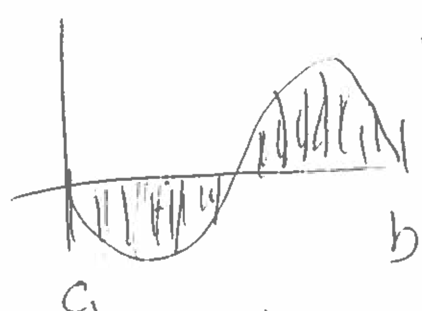
Ex

$$\int_0^{t} x\sin(x^2)\,dx = g(t)$$

$$t\sin(t^2) = g'(t)$$

$$\frac{d}{dt}\left(F(t) - F(0)\right)$$

$$f(t) + 0$$



$$\int_a^b h(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$F(b) - F(a) = -\left(F(a) - F(b)\right)$$

$$\int_a^a f(x)\,dx$$

$$F(a) - F(a) = 0$$

<u>Ex</u> $\displaystyle\int_{-5}^{5} x\sin(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int_{u(-5)=25}^{u(5)=25} \frac{1}{2}\sin(u)\,du = 0$$

Ex

$$\int x^3 e^{x^4}\,dx$$

$u = x^4$

$du = 4x^3\,dx$

$$\int x^4 \sqrt{x^5+10}\,dx$$

$$\int x^3 \sin(x^4)\,dx$$

$u = x^4$

$du = 4x^3\,dx$

$$\int \frac{1}{1+x^2}\,dx$$

$\tan^{-1}(x)+C$

$$\int \frac{x}{1+x^2}\,dx$$

$u = 1+x^2$

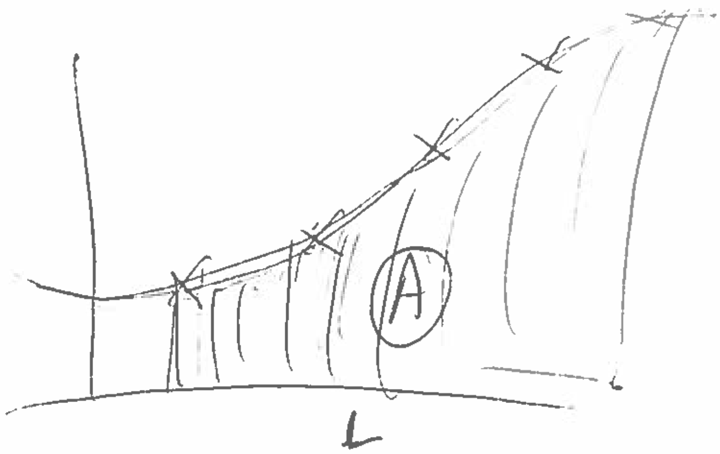
$du = 2x\,dx$

$$\int \frac{1}{u}\,\frac{du}{2}$$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u|+C$

$\frac{1}{2}\ln(1+x^2)+C$



$\frac{A}{L}$ = Ave. Value

$$\int \frac{x^2}{1+x^2}\,dx = \int 1 - \frac{1}{x^2+1}\,dx$$

$$1 - \frac{1}{x^2+1}$$

$$x^2+1 \,\overline{)\, x^2}$$

$$-(x^2+1)$$

$$-1$$

$$\int 1\,dx - \int \frac{1}{x^2+1}\,dx$$

$$= x - \tan^{-1}(x) + C$$