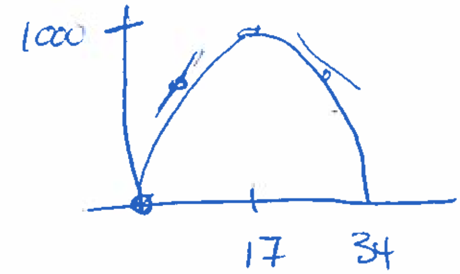
Initial Value Problems



miles : $S(t)$ = Position at Time t

miles/hr : $S'(t)$ = Velocity $= V(t)$

miles/hr/hr $= S''(t)$ = Acceleration $= V'(t)$

Ex acceleration of gravity is -9.8 m/sec

If I threw a ball UP at 6 m/sec ⟵ initial value

Then what's the speed after 4 sec?

$V(0) = 6$

$V_0 = 6$

$$V(t) = \int a(t)\,dt$$
$$= \int -9.8\,dt = -9.8t + C$$

$$V(t) = -9.8t + C \rightarrow V(0) = \cancel{(-9.8)(0)} + C = 6$$

velocity at any t. ⟶ $V(t) = -9.8t + 6$

$$V(4) = -9.8(4) + 6 = -33.2 \text{ m/sec}$$

Speed 33.2 m/sec

Ex What if my initial height of ball throwing is 2 meters. Find an equation for position $S(t)$

$$S(t) = \int v(t)\,dt = \int -9.8t + 6\,dt$$

$$S(t) = -4.9t^2 + 6t + C \qquad S(0) = 2$$

$$S(0) = 0 + 0 + \underline{C = 2}$$

$$S(t) = -4.9t^2 + 6t + 2$$

Ex $a(t) = t^2 + \cos t \qquad V(0) = 2 \qquad S(1) = 8$

Find $S(t) = ?$

$$V(t) = \int a(t)\,dt = \int t^2 + \cos t \; dt = \frac{t^3}{3} + \sin(t) + C$$

$$V(0) = 0 + \sin(0) + \boxed{C = 2} \qquad V(t) = \frac{t^3}{3} + \sin(t) + 2$$

$$S(t) = \int V(t)\,dt = \int \frac{t^3}{3} + \sin(t) + 2 \; dt = \frac{t^4}{12} - \cos(t) + 2t + C$$

$$S(1) = \frac{1}{12} - \cos(1) + 2 + C = 8$$

$$C = 8 - 2 - \frac{1}{12} + \cos(1)$$

$$S(t) = \frac{t^4}{12} - \cos(t) + 2t + 5\tfrac{11}{12} + \cos(1)$$

$a(t) = e^t \qquad V(t) = \int e^t dt = e^t + C \qquad V(0) = 2$

$V(0) = e^0 + C = 2$

$1 + C = 2 \qquad C = 1$

$$\frac{d}{dx} e^{x^2} = e^{x^2} \cdot (2x)$$

$$\Leftarrow \int e^{x^2}\, 2x\, dx$$

Substitution $\quad u = x^2$

$du = 2x\,dx$

$$\int e^u \cdot du = e^u + C$$

$u = x^2 \qquad = e^{x^2} + C$

Ex $\displaystyle\int \sin(4x)\,dx = \int \sin(u)\frac{du}{4} = \frac{-\cos(4x)}{4} + C$

$u = 4x$

$du = 4\,dx$

$\frac{du}{4} = dx$

$\frac{1}{4}\int \sin(u)\,du$

$\frac{-\cos(u)}{4} + C$

Ex $\displaystyle\int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C$

denominator $= u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= -\displaystyle\int \frac{1}{u}\,du = -\ln|u| + C$

Ex $\displaystyle\int x(x+1)^{3/7}\,dx$

$u = x+1 \rightarrow x = u-1$

$du = dx$

$\displaystyle\int (u-1)(u^{3/7})\,du = \int u^{10/7} - u^{3/7}\,du$

$\frac{7u^{17/7}}{17} - \frac{7u^{10/7}}{10} + C$

$= \frac{7}{17}(x+1)^{17/7} - \frac{7}{10}(x+1)^{10/7} + C$