151 dis

Calculus = Change

Average Rate $\frac{\Delta y}{\Delta x}$ 2 points

Instantaneous Rate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

1 point.

↓

Slope of Tangent Line

↓

Derivative.

Rules for finding derivatives.

Chain Rule

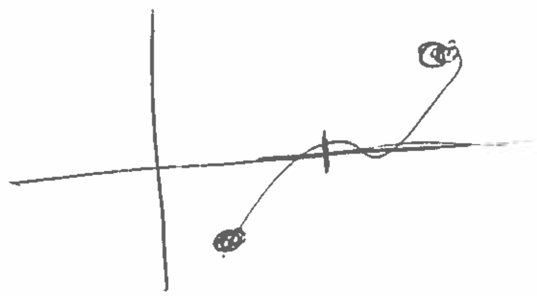
- Implicit Differentiation
- Logarithmic Differentiation
- Related Rates. (Balloon)
- L'Hopital's (used for Limits)
- Analysis of Functions
- Optimization (Today)

Theorems

Squeeze Theorem $\lim_{x \to \infty} \frac{\sin x}{x}$

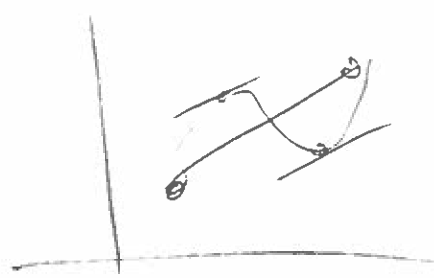
Intermediate Value Theorem $\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$

$f(x)$ continu



Mean Value Theorem

Instantaneous = Ave Rate
Somewhere



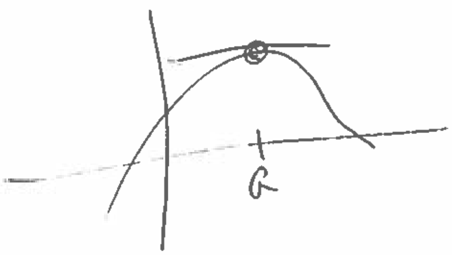
Given a Function

Max or mins occur at

critical numbers

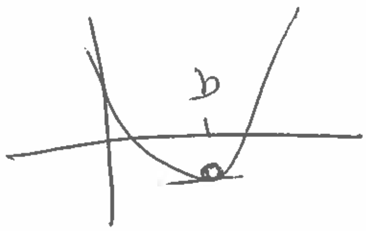
= when $f'(x) = 0$ or

$f'(x)$ is undefined



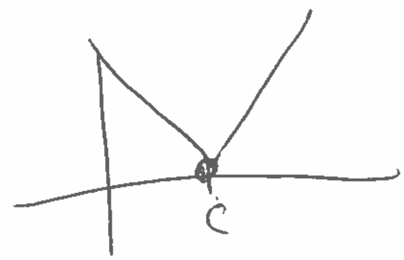
MAX

$f'(a) = 0$



MIN

$f'(b) = 0$



MIN

$f'(c) = \text{UND}$

1st Derivate Test

		a	
$f'(x)$	+	0	−
	Inc		DEC

MAX by 1st Deriv Test

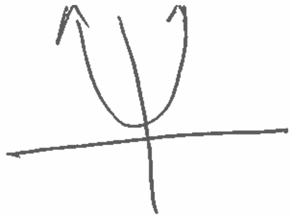
	b	
−	0	+
DEC		INC.

MIN by 1st Deriv Test

	c	
−	und	+

Could be a MIN.

Ex $y = x^4$



$y' = 4x^3$

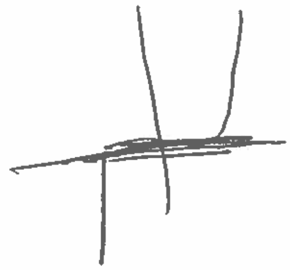
$y' = 4x^3 = 0$

when $x = 0$

$y'' = 12x^2$

$y''(0) = 0$ Inconclusive

$y = x^3$



$y' = 3x^2 = 0$

$x = 0$

y'	+	0	+
	INC	↑	INC

NOT MAX or MIN

Second Derivative TEST

If x_1, x_2, x_3 are critical Numbers

and If $f''(x_1) > 0$ — **MIN**

+ concave Up

and If $f''(x_1) < 0$ — **MAX**

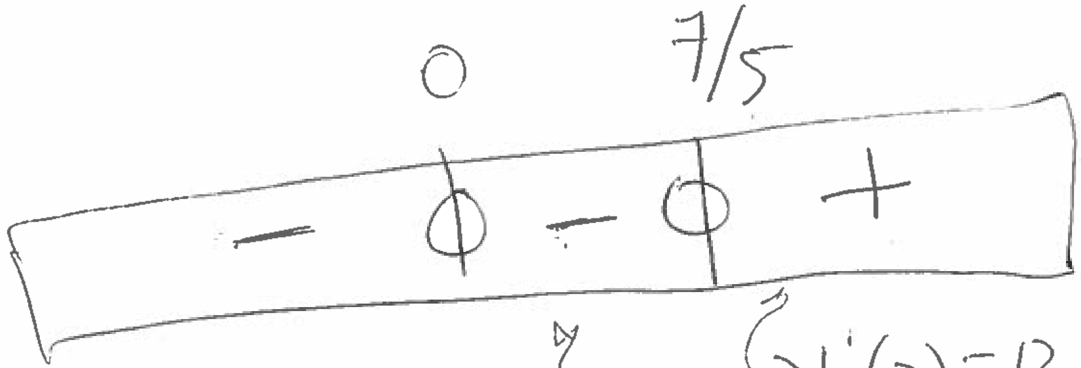
− concave Down

and If $f''(x_1) = 0$

Inconclusive

$\underline{Ex}$ $y' = x^2(5x-7)$

zeros (critical numbers)



$y'(2) = 12$ $+$

criticals: $0, 7/5$

$y'(1) = -2$ $-$

$y'(-1) = -12$

$7/5$

$-$ | $+$

MIN by 1st deriv. Test

$y' = 5x^3 - 7x^2$

$y'' = 15x^2 - 14x$

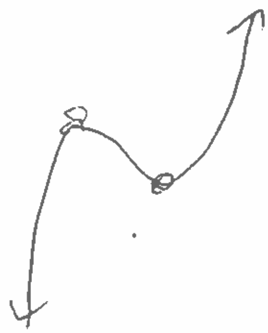
$y''(0) = 0$ inconclusive

$y''(7/5) = 15\cdot\left(\frac{7}{5}\right)^2 - 14\left(\frac{7}{5}\right) = 9.8$ $+$

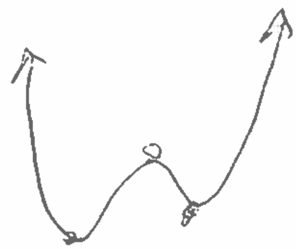
MIN by 2nd Der Test

CCW UP

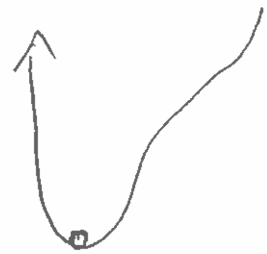
Cubic or Quartic.



0 crit.

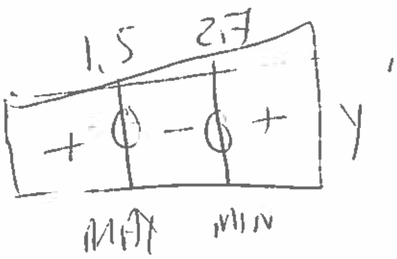


3 criticals



1 critical

2 critica



$y''(2.7) = \boxed{+}$

$y''(1.5) = \boxed{-}$

" Lady G. ~~MAX~~ MIN Rev. at ~~$~~ $5.2

by second deriv test "

Critical at 5.2.

$y''(5.2) = +8.22$ concave UP

MIN

GROUP NAME: El Business

Date: 3/25

Student Names (First and Last)

Speaker/Presenter: Ryan

Writer/Prep: Brittany Bayo

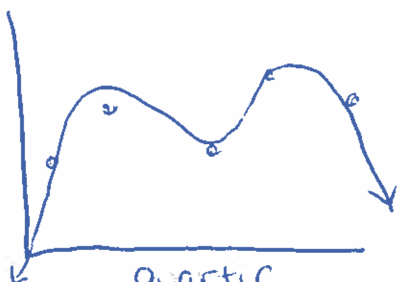
Leader/Collaborator: ______

Independant Variable (x-axis): world cups

Dependant Variable (y-axis): goals scored

Conclusion (in words): Goals scored were highest in the second world cup by the second derivative.

Supporting Work:



quartic

1st $x = 1.49$ critical numbers or zeros

$x = 2.88$

$x = 4.54$

1.49		2.88		4.54	
−	ϕ	+	ϕ	+	ϕ −
	min		inc		max

2nd

inflection don't care

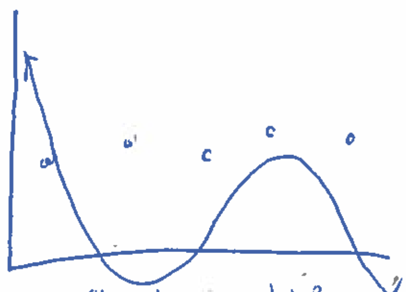
$x = 1.49$ (−)

$x = 2.88$ (+) ← inflection, don't care

$x = 4.54$ (−)

concave down ∩

maximum by second derivative



1st derivative

$y''(1.49) = -51$ MAX

$y''(2.88) = +27$ MIN

$y''(4.54) = -60$ MAX

GROUP NAME: Squiggles Я Us

Date: 3/25/14

Independant Variable (x-axis): hours of party

Dependant Variable (y-axis): liters of alcohol consum[illegible]

Student Names (First and Last)

Speaker/Presenter: Kevin V

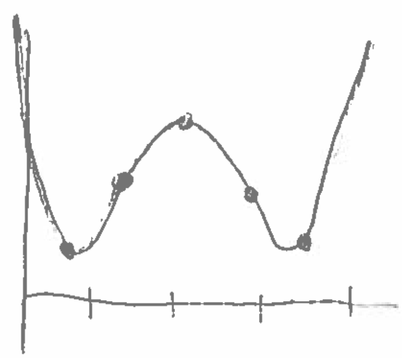
Writer/Prep: Anik Patel

Leader/Collaborator: Kevin I

Conclusion (in words): The peak of the party is [illegible] about [illegible]

Supporting Work:

X	Y
1	30
2	55
3	70
4	45
5	32



a = 3.416
b = [illegible].66
c = 144.583
d = 185.833
e = 107
$r^2 = 1$

min: x = 1.005, y = 24.[illegible]

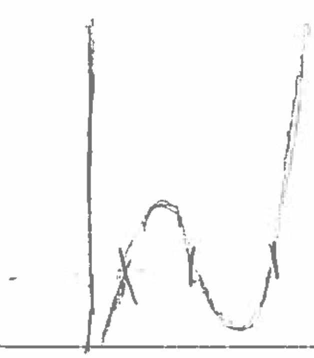
max: x = 2.856, y = 70.494

min: x = 4.737, y = 28.189

y'' (95.978) ⊕ MIN

$y''(1.005) = + $ MIN $y''(2.856) = -47.57$ MAX

$y_1 = 3.4166666666585x^4 + {}^-39.1666666656 9x^3 + 144.58333x^2$ [illegible]



Crit. ~~.75~~ 1.8 2.1

GROUP NAME: I ❤ cence

Date: 3/25

Student Names (First and Last)

Speaker/Presenter: Lindsey Lundberg

Writer/Prep: Corrina Hansen

Leader/Collaborator: ______

Independant Variable (x-axis): Time (Hours)

Dependant Variable (y-axis): Drug Concentration (ppm)

Conclusion (in words): According to the cubic regression, the drug concentration reached a minimum at 6 hours, by the second derivative test.

Supporting Work:

$$f(x) = 0.4629\ldots x^3 - 3.2936\ldots x^2 - 12.4074\ldots x + [illegible]$$

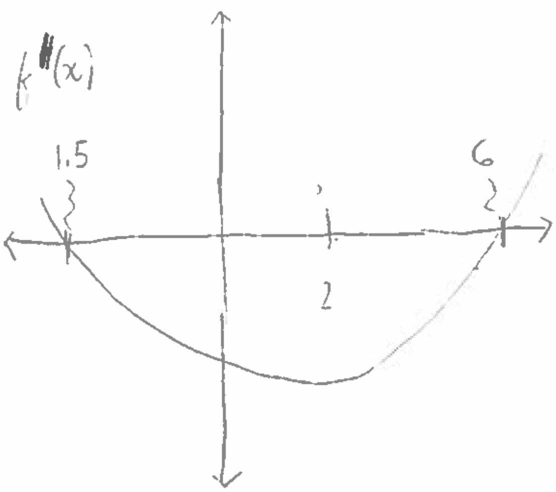
$$f'(x) = 3(0.4629\ldots)x^2 - 2(3.2936\ldots)x - 12.4074 = 0$$

$$f''(x) = 6(0.4629\ldots)x - 2(3.2936\ldots)$$

crit.

$x = -1.5$

$x = 6$



$f''(-1.5) = \ominus$ MAX

$f''(6) = \oplus$ MIN

	-1.5		0		6	
$y' =$	−	0	+	0	−	0	+

	1.5		0		6	
$y'' =$	+	0	−		−		+

↑ MAX ↑ MIN

GROUP NAME: FA - LA - LE - DA BLAH - BLAH

Date: 25MAR14

Independant Variable (x-axis): # of correct assinments done

Dependant Variable (y-axis): time in minutes

Student Names (First and Last)

Speaker/Presenter: Greg McAvoy

Writer/Prep: Keith Meserll

Leader/Collaborator: Harrison Smith

Conclusion (in words): When we do -.3851, we do none of the work.

Supporting Work:

$O_1 = -.3851064$

X	Y_2	Y_3
-0.3851	-.3687	9.9799

$Y^3 = 9.9799$ (+)

min

Quartic Reg

$y = ax^4 + bx^3 + cx^2 + dx + e$

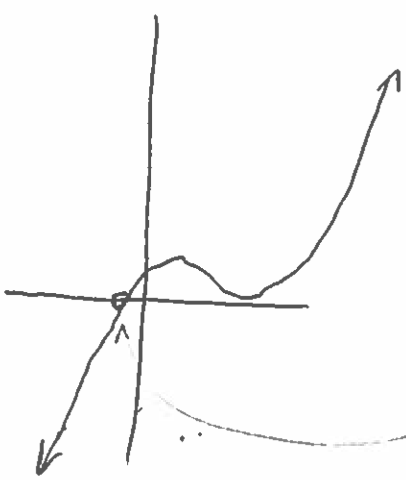
$a = 0.0523809524$

$b = -0.8309523809$

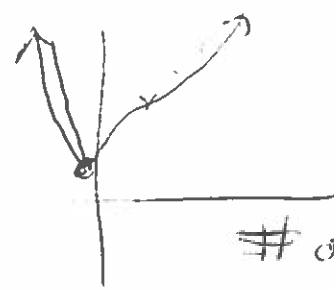
$c = 3.98\overline{3}$

$d = 3.08095238$

$e = 3.71428571$



X = -.3851



# of assynmt

X	Y
1	10
2	20
4	40
6	54
9	42

$Y_2(-.3851) = 9.9799$ (+)

MIN

GROUP NAME: I ♡ Shoes

Date: 3/26/14

Independant Variable (x-axis): Years

Dependant Variable (y-axis): Shoes

Student Names (First and Last)

Speaker/Presenter: Jae Sinclair

Writer/Prep: Dominique C-

Leader/Collaborator:

Conclusion (in words):

In middle of 2011 we sold the ~~least~~ most paires of shoes ~~sold~~

Supporting Work:

QUARTIC REGRESSION

$X = 11.46, 13.85, 11.27$

$Y_1 = .045$

$Y_2 = 78.18$

$\boxed{Y_3 = 2.70}$ – SECOND DERIVITAVE TEST

DATA

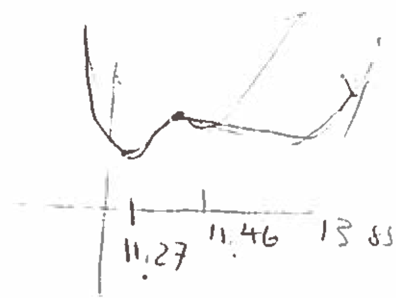
10	63
11	78
12	79
13	87.5
14	94.3

ZEROS

$X = 11.46 \rightarrow -.0041 \rightarrow$ MAX $-$

$X = 13.85 \rightarrow .46029 \rightarrow$ MIN $+$

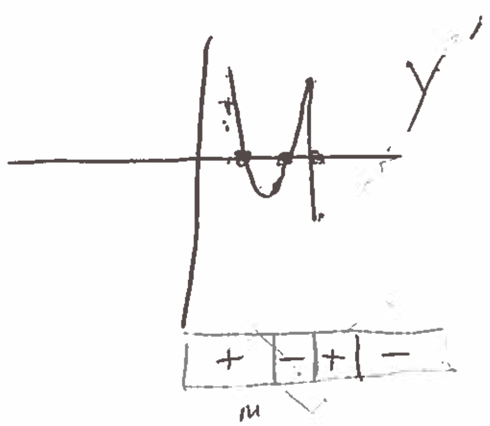
$X = 11.27 \rightarrow .01155 \rightarrow$ MIN $+$



min	max	min	
−	+	−	+

11.27 11.46 13.85

critical numbers



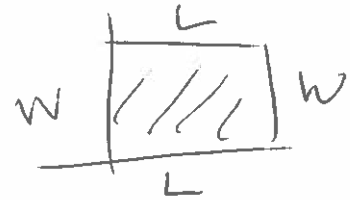
TSIR
d15

OPTIMIZATION

Ex Suppose

$$A = L \times W = A(L, W)$$

Maximize area



Constraint might be

50' of Fencing = L + W + L + W

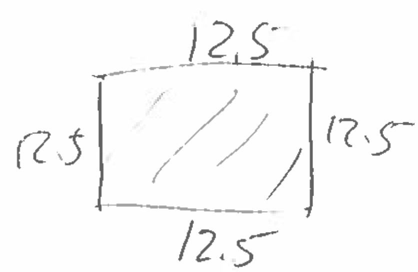
$$50 = 2L + 2W$$

$$25 = L + W$$

$$W = 25 - L$$

$$A(L, W) = A = L \times W$$

$$A(L) = L(25 - L)$$



$$A = 25L - L^2$$

$$A' = 25 - 2L = 0$$

$$25 = 2L \qquad L = 12.5$$

Given $f(x, y, z)$

will need 2 constraints

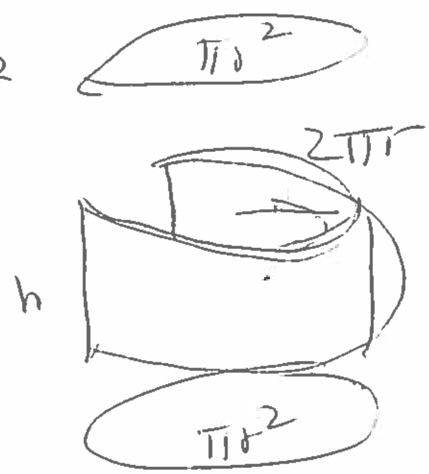
Process

1. Identify Function to be optimized or max. or min.
2. If its a function of multiple variables, use constraints to reduce to 1 variable.
3. Find max/min

Maximize Volume given fixed amount of material 100 in²

Volume

$$V(r,h) = \pi r^2 \cdot h$$



$$\text{Area} = \pi r^2 + 2\pi r h + \pi r^2$$

$$100 \text{ in}^2 = 2\pi r^2 + 2\pi r h$$

$$\frac{100 - 2\pi r^2}{2\pi r} = h$$

$$V(r) = \pi r^2 \cdot \left(\frac{100 - 2\pi r^2}{2\pi r} \right)$$

$$= 50r - \pi r^3$$

$$V' = 50 - 3\pi r^2 = 0$$

$$50 = 3\pi r^2$$

$$\frac{50}{3\pi} = r^2$$

$$\sqrt{\frac{50}{3\pi}} = r$$

$$= r$$

$$2.30$$

$$h = \frac{100 - 2\pi(1.78)^2}{2\pi(1.78)} = 4.60$$

4.6

4.6