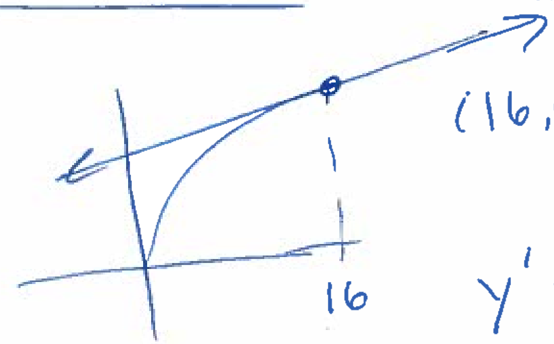
Applications of Derivatives

Linearization of Function

$y = \sqrt{x}$

$\sqrt{17} \approx ?$



$y' = \frac{1}{2}x^{-1/2}$

$y'(16) = \frac{1}{8}$

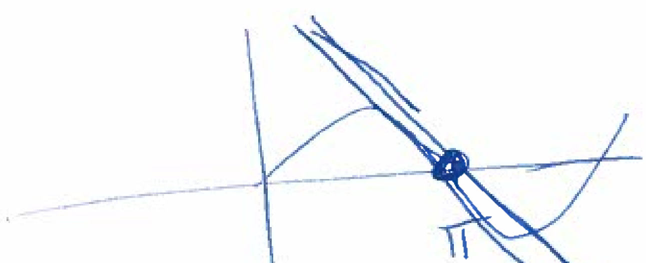
$y - 4 = \frac{1}{8}(x - 16)$

when (17, y)

$y - 4 = \frac{1}{8}(17 - 16)$

$y = 4\frac{1}{8}$

Approximate $\sin\left(\frac{11}{4}\right)$ $\qquad \frac{11}{4} \approx \pi$



$m = f'(\pi) = \cos(\pi) = -1$

Point $(\pi, 0)$

$y = -(x - \pi)$

$y\left(\frac{11}{4}\right) = -\left(\frac{11}{4} - \pi\right)$

0.39159

Newtons Method

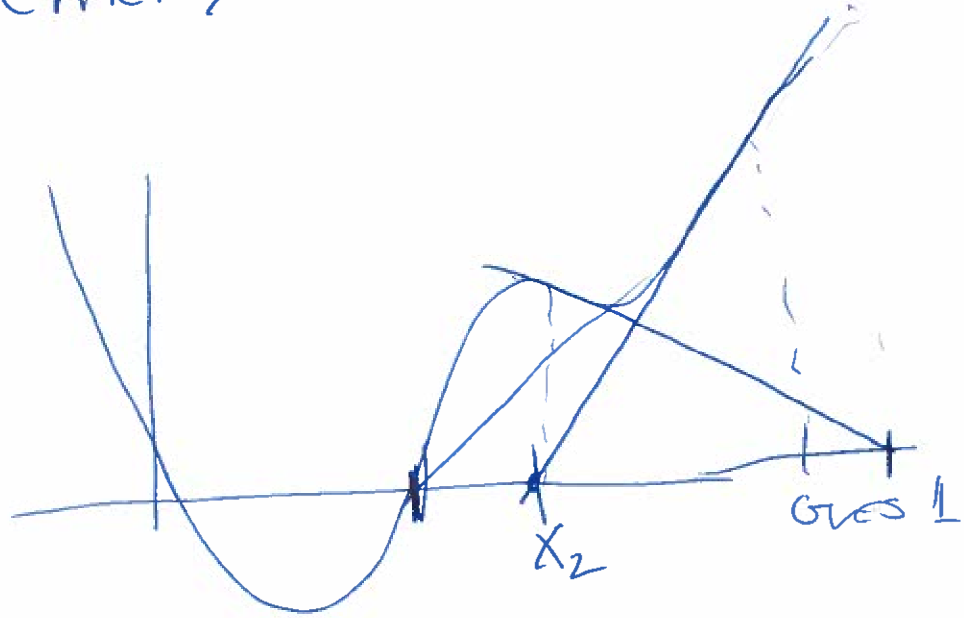
$x^4 - 3x^2 - 7x + 8 = 0$ Guess $x = 1$

(1) $Y_1 = x \wedge 4 - 3x^2 - 7x + 8$

(2) $1 \xrightarrow{STO} X$

(3) $X - Y_1 / \text{nderiv}(Y_1, X, X) \xrightarrow{STO} X$

<enter> ...... until settles on.



Related Rates

$$E = mc^2$$

$$\frac{d}{dt}E = \frac{d}{dt}mc^2$$

$$\frac{dE}{dt} = m \cdot \frac{d}{dt}c^2 + c^2 \cdot \frac{d}{dt}m$$

$$\frac{dE}{dt} = m \cdot 2c\frac{dc}{dt} + c^2\frac{dm}{dt}$$

Rates	Vales
$\frac{dE}{dt} =$	$m = 50$
$\frac{dc}{dt} = 0$	$c = 3 \times 10^8$
$\frac{dm}{dt} = +6/hr$	

c is constant @ 3×10^8

m is 50 and growing by 6 per hr.

L'Hôpitals Rule   LHR

Low Pee Tal

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$ use LHR

$$\Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Ex

$$\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$$ use LHR

$$\lim_{x \to 0} \frac{\cos x}{1} = \cos(0) = 1$$

Ex

$$\lim_{x \to \infty} \frac{3x^2 - 7x}{4x^2 + 5} = \frac{\infty}{\infty}$$ use LHR

$$\lim_{x\to\infty} \frac{6x-7}{8x} = \frac{\infty}{\infty} \text{ use (LHR)}$$

$$\lim_{x\to\infty} \frac{6}{8} = \frac{6}{8} \text{ or } \frac{3}{4}$$

$$\lim_{x\to\infty} \frac{3}{x^{320}} \cdot x^{320} = 3$$

$$\frac{3}{x^{320}} = 0, \quad x^{320} = \infty$$

$0 \cdot \infty$ = could be anything

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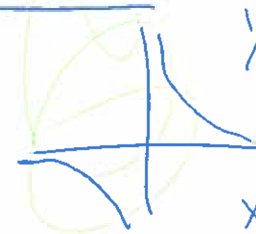
0   1 ∞



Multiply

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$0 \cdot \infty = CBA$
$1 \cdot \infty = \infty$
$1 \cdot 1 = 1$
$\infty \cdot \infty = \infty$

Division

$0/0 = CBA$
$0/1 = 0$
$0/\infty = 0$
$1/0 = \pm\infty$
$1/\infty = 0$
$\infty/0 = \pm\infty$
$\infty/\infty = CBA$

Add - No problems

Subtraction

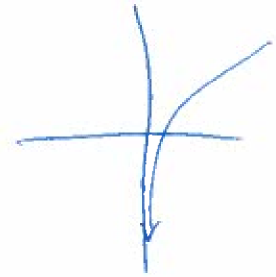
$1 - \infty = -\infty$
$\infty - \infty = CBA$

Exponents

$0^0 = CBA$
$1^\infty = CBA$, $.9^\infty = 0$
$1.01^\infty = \infty$
$1^\infty = 1$

Ex: $\lim_{x \to 0^+} x^2 \ln x$

$(0 \cdot (-\infty))$



$$\lim_{x \to 0^+} \frac{\ln x}{x^{-2}} = \frac{\infty}{\infty} \quad \text{use (LHR)}$$

$$\lim_{x \to 0^+} \frac{1/x}{-2x^{-3}} \quad \text{use algebra}$$

$$\lim_{x \to 0^+} -\frac{x^3}{2x} = \lim_{x \to 0^+} -\frac{1}{2}x^2 = 0$$

Ex

$$P = Q\left(1 + \frac{R}{N}\right)^{NT}$$

compound Interest

N = # compounds per yr.

$\downarrow$

$$P = Qe^{RT}$$

compound continuously

$$\lim_{N\to\infty}\left(1+\frac{R}{N}\right)^N =$$

Remember $e^{\ln x} = x$

$$= e^{\ln \lim_{N\to\infty}\left(1+\frac{R}{N}\right)^N}$$ ← Property of Logs/Inverses

$$= e^{\lim_{N\to\infty} \ln\left(1+\frac{R}{N}\right)^N}$$ ← Property of Limits

$$= e^{\lim_{N\to\infty} N\cdot\ln\left(1+\frac{R}{N}\right)}$$ ← Property of Logs. "Ladder" prop

$0 \cdot \infty$

$$= e^{\lim_{N\to\infty} \frac{\ln\left(1+\frac{R}{N}\right)}{N^{-1}}} = \frac{0}{0}$$

use LHR

$$= e^{\lim_{N\to\infty} \frac{\frac{1}{\left(1+\frac{R}{N}\right)}\cdot\frac{d}{dN}\left(1+\frac{R}{N}\right)}{\frac{d}{dN}\left(\frac{1}{N}\right)}}$$

$$= e^{\lim_{N \to \infty} \frac{\frac{1}{(1+\frac{R}{N})} \cdot \left(0 + R \cdot \frac{d}{dN}\frac{1}{N}\right)}{\frac{d}{dN}\frac{1}{N}}}$$

$$= e^{\lim_{N \to \infty} \frac{R}{1+R/N}} = e^{\frac{R}{1+0}} = e^{R}$$

$$\infty - \infty \rightarrow$$

Ex $$\lim_{x \to \infty} \left(\sqrt{x} - \sqrt{1+x}\right) \cdot \frac{\sqrt{x} + \sqrt{1+x}}{\sqrt{x} + \sqrt{1+x}}$$

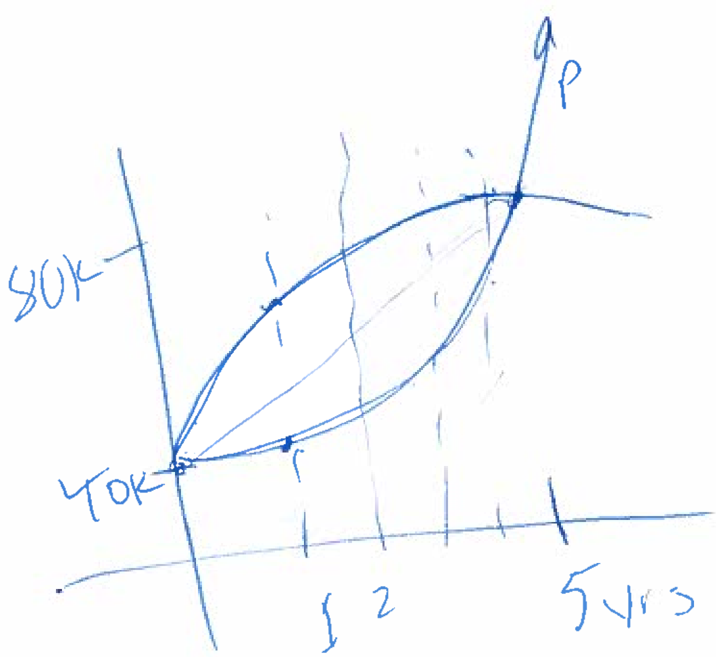
$$\lim_{x \to \infty} \frac{x - 1 - x}{\sqrt{x} + \sqrt{1+x}} =$$

$$\lim_{x \to \infty} \frac{-1}{\sqrt{x} + \sqrt{1+x}} = \frac{-1}{\infty} = 0$$

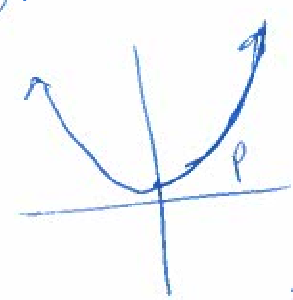
Second Derivative

Function y'' $\frac{d^2y}{dx^2}$ D_{xx} $f''(x)$

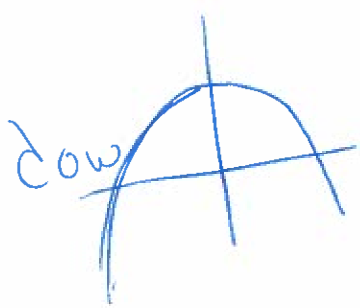
Values $y''(a)$ $\left.\frac{d^2y}{dx^2}\right|_{x=a}$ $D_{xx}(a)$ $f''(a)$



Concave Up



Concave Down



X	y
1	10
2	20
	40
3	100
4	120
5	

Regression Not Linear / Quad.

Y_1 = Cubic Regression
VARS 5 > > 1.

$Y_2 = \text{nderiv}(Y_1, X, X)$

$Y_3 = \text{nderiv}(Y_2, X, X)$

In 5th year, while Lady G was increasing in value, Her futures were slowing down leveling off or, not looking as good

GROUP NAME: I ♡ Shoes

Date: 3/11

Independant Variable (x-axis): Years (2010-2014)

Dependant Variable (y-axis): Salary (65k - 85k)

Student Names (First and Last)

Speaker/Presenter: Dominique C.

Writer/Prep: Val Sinclair

Leader/Collaborator:

Conclusion (in words):

~~[illegible]~~, in 2015, we will have a yearly salary of $90,000.

Supporting Work:

x	y
10	65
11	70
12	75
13	80
14	85

y1=0x^3+0x^2+5x+15

y2= nDeriv (y1, x, x)

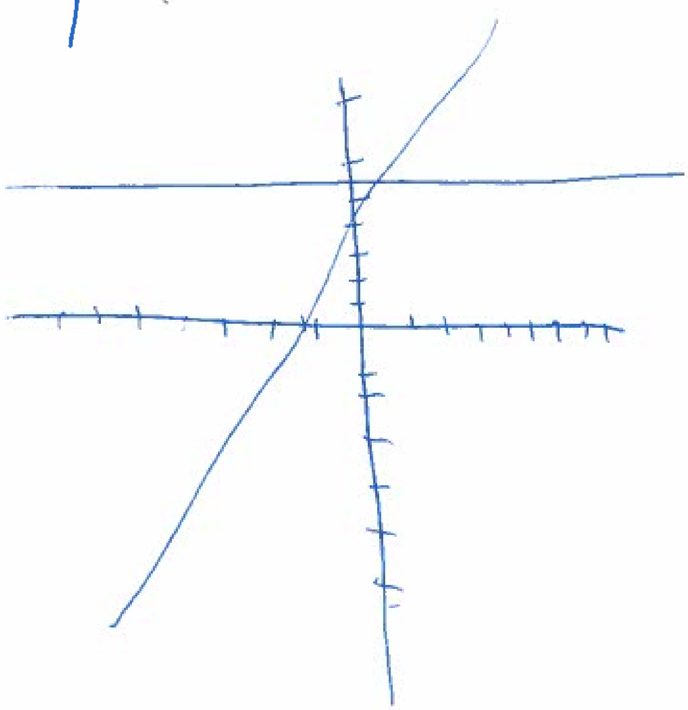
y3= nDeriv (y2, x, x)

$y = 5x+15$

$y' = 5$

$y'' = 0$

increasing value of 5,000 per year.



GROUP NAME: El Business

Date: 3/11/14

Independant Variable (x-axis): World Cups (in Yrs)

Dependant Variable (y-axis): Goals Scored

Student Names (First and Last)

Speaker/Presenter: RYAN

Writer/Prep: ANDY

Leader/Collaborator: BRITTANY

Conclusion (in words): In the 3rd world cup, while goals scored were increasing at 3.333 goals per world cup, they ~~[scribbled out]~~ continued increasing at an ~~[scribbled out]~~ increasing rate.

Supporting Work:

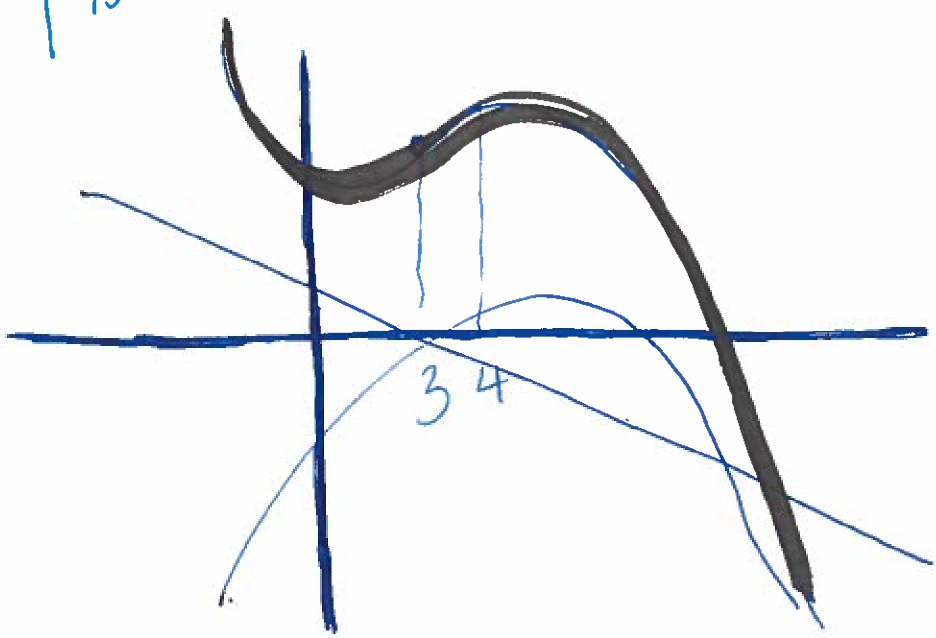
X	Y
1	35
2	39
3	31
4	45
5	43

Cubic regression [STAT] (>) 6:

$Y_1 =$ [VARS] 5 (>) (>) 1:

1ST DERIV. $Y_2 = \text{nderiv}(y_1, x, x)$ [MATH] 8: [VARS] (>) 1:

2ND DERIV. $Y_3 = \text{nderiv}(y_2, x, x)$ [MATH] 8: [VARS] (>) 2:



GROUP NAME: The Engineering Group Because My Names are Lame Apparently.	Student Names (First and Last)
Date: 11 MAR14	Speaker/Presenter: Greg McAvoy
Independant Variable (x-axis): year	Writer/Prep: Keith Meserall
Dependant Variable (y-axis): # in thousands	Leader/Collaborator: Harrison

Conclusion (in words): In in year 12 the # of solder in Ukraine is 4,2043 thousand and the $\frac{d^2y}{dx^2}$ is -1,242

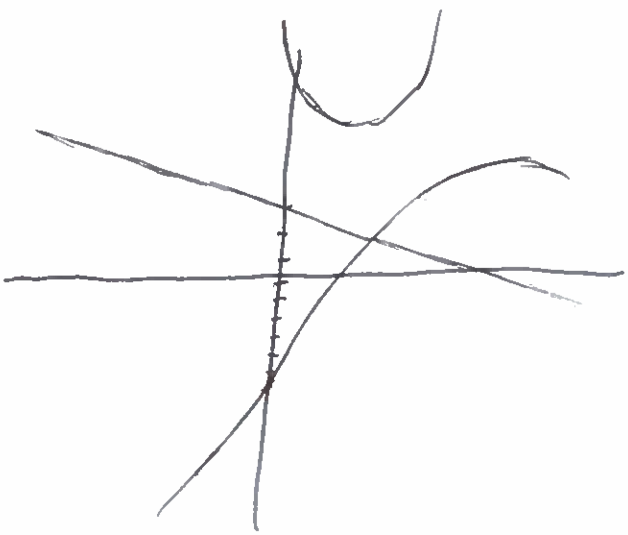
Supporting Work:

x	y
3	17
6	28
7	39
9	41
10	54
14	70

$Y_1 = -0.0555300894524x^3 + 1.3783085325909x^2 + -4.886074222 0912x + 20.8601734 28638$

$Y_2 = \text{nDeriv}(Y_1, X, X)$

$Y_3 = \text{nDeriv}(Y_2, X, X)$



~~X | Y1 | Y2 | Y3~~
~~12 | 70.224 | 1.6544 | -1.908~~

X	Y_1	Y_2	Y_3
12	64.7848	4.2043	-1.242
yr	people	people/yr	peo/yr/yr

GROUP NAME: ♡ Science

Date: 11MAR14

Independant Variable (x-axis): Time (hr)

Dependant Variable (y-axis): Drug Concentration (PPM)

Student Names (First and Last)

Speaker/Presenter: Corrina H

Writer/Prep: Lindy

Leader/Collaborator:

Conclusion (in words):

In the 3rd hour drug concentration is decreasing by less ~~[illegible]~~ speeding ~~[illegible]~~

Supporting Work:

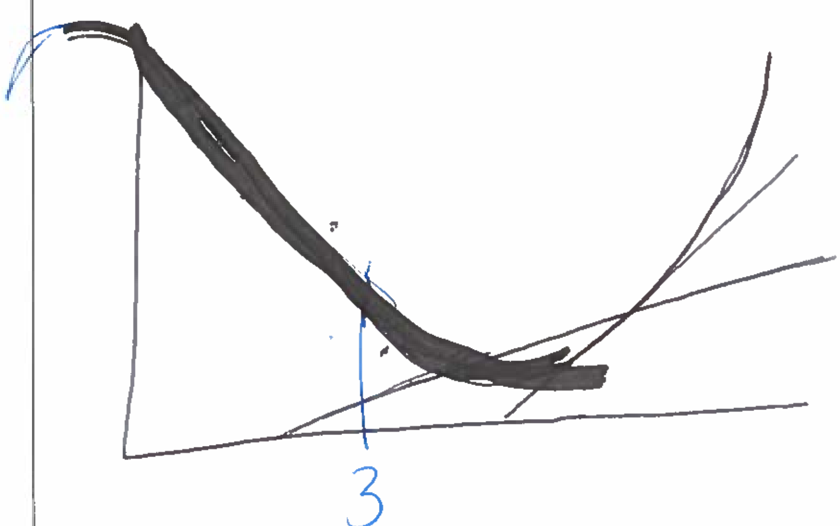
Time (hr)	Drug Conc. (PPM)
0	100
1	85
2	60
3	55
4	20
5	15

Cubic Reg

$y_1 = .463...x^3 + 3.29...x^2 - 12.407...x + 99.68...$

$Y_2 = \text{nDeriv}(Y_1, X, x)$

$Y_3 = \text{nDeriv}(Y_2, X, x)$



X	Y_1	Y_2	Y_3
3	45.317	-19.67	1.746
Time	PPM	PPM/hr	PPM/hr/hr

GROUP NAME: Squiggles Я Us

Date: 3/11/2014

Independant Variable (x-axis): Time in hours

Dependant Variable (y-axis): liters of alcohol drunk

Student Names (First and Last)

Speaker/Presenter: Mishelle

Writer/Prep: Anik Patel

Leader/Collaborator: Kevin V

Trainee : Kevin I

Conclusion (in words):

The party is ending very quickly at 6 hours

Supporting Work:

X	Y
1	40
2	100
3	200
4	246
5	290

CubicReg

$y=ax^3+bx^2+cx+d$

$a=-3.33...$

$b=23.92857143$

$c=22.26190476$

$d=-5$

$Y_1 = -3.33...x^3 + 23.92857x^2 + 22.2619x + -5$

$Y_2 = \text{nDeriv}(Y_1, x, x)$

$Y_3 = \text{nDeriv}(Y_2, x, x)$

hr's	liters drunk	liters drunk'	
X	Y_1	Y_2	y_3
6	270	-50.6	-72.14
7	180	-132.7	-92.14
8	-2.143	-234.9	-112.1
9	-296.4	-357	-132.1
10	-722.9	-499.2	-152.1