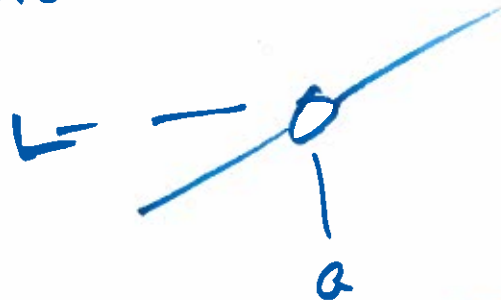


$$\lim_{h \rightarrow 0} \frac{h}{h} (2x+h) = 2x$$

↑
has a "hole" at $h=0$



ϵ - δ definition of Limit

ϵ error
 δ delta

Given any $\epsilon > 0$ (usually very small)
There exists a $\delta > 0$ so that
whenever $|x - a| < \delta$ then

$$|f(x) - L| < \epsilon$$

$$\lim_{x \rightarrow a} f(x) = L$$

Find δ given $\epsilon = .01$,

Verify ϵ - δ definition of Limit.

$$\lim_{x \rightarrow ?} 3x + 5$$

$$= 11$$

$$\| f(x) - L \| < \epsilon$$

$$\| 3x + 5 - 11 \| < .01$$

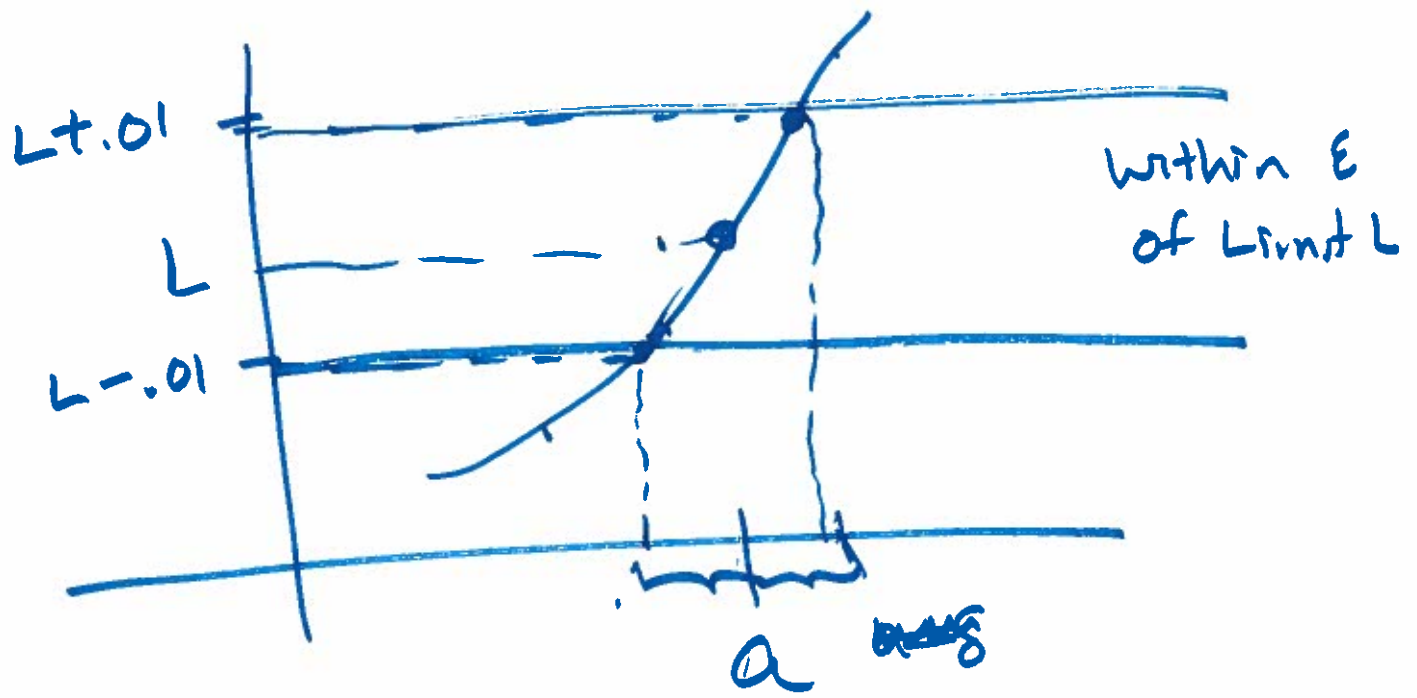
$$\| 3x - 6 \| < .01$$

$$3 \| x - 2 \| < .01$$

$$\| x - 2 \| < \frac{.01}{3} \approx .003...$$

$$\| x - a \| < \delta$$

$$\begin{array}{l} 2.003 \\ 1.997 \end{array}$$



$$E = .01$$

$$a - \delta \quad a + \delta$$

$$1.8 - 2 - 2.1$$

.2 $\textcircled{!}$ Smaller

$$2 - .1 \quad 2 + .1$$

$$1.9 \quad 2.1$$

Good range

2nd Trace S : Intersect

1st Curve : Y_1

2nd Curve : Y_2

Guess : 2

$$X = 2.00333\ldots \quad Y = 11.01$$

$$S = .00333\ldots$$

2nd Trace S : Intersect

1st Curve : Y_1

2nd Curve : $\textcircled{V} Y_3$

Guess : 2

$$X = 1.996666\ldots \quad Y = 10.99$$

You do This

$$Y_1 = \text{reg1} / (X \leq a) + \text{something}$$

$$Y_2 = \text{reg2} / (X \geq a)$$

$$Y_3 = L + \epsilon(.01)$$

$$Y_4 = L - \epsilon(.01)$$

Calc 5: Intersect

1st curve Y_1

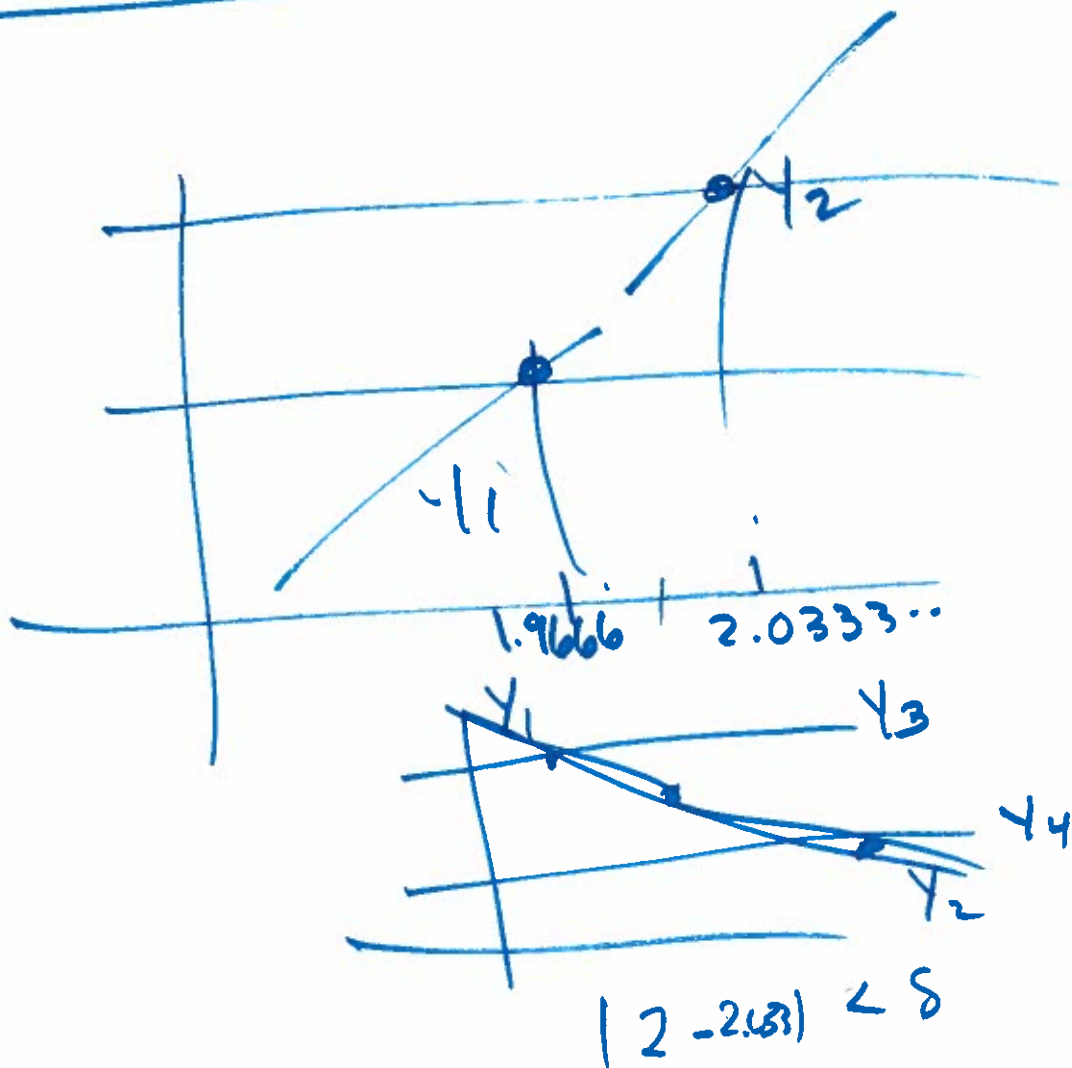
2nd curve: Y_3 or Y_4

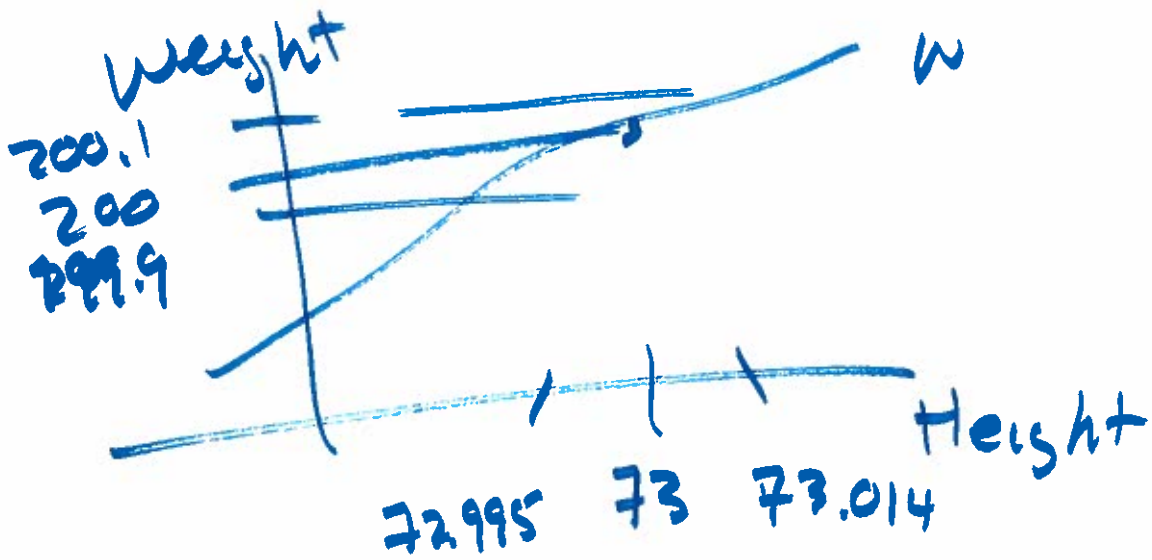
Guess.

Calc 5: Intersect

1st curve Y_2

2nd curve: Y_4 or Y_3





As long as the height is between
72.995 and 73.014

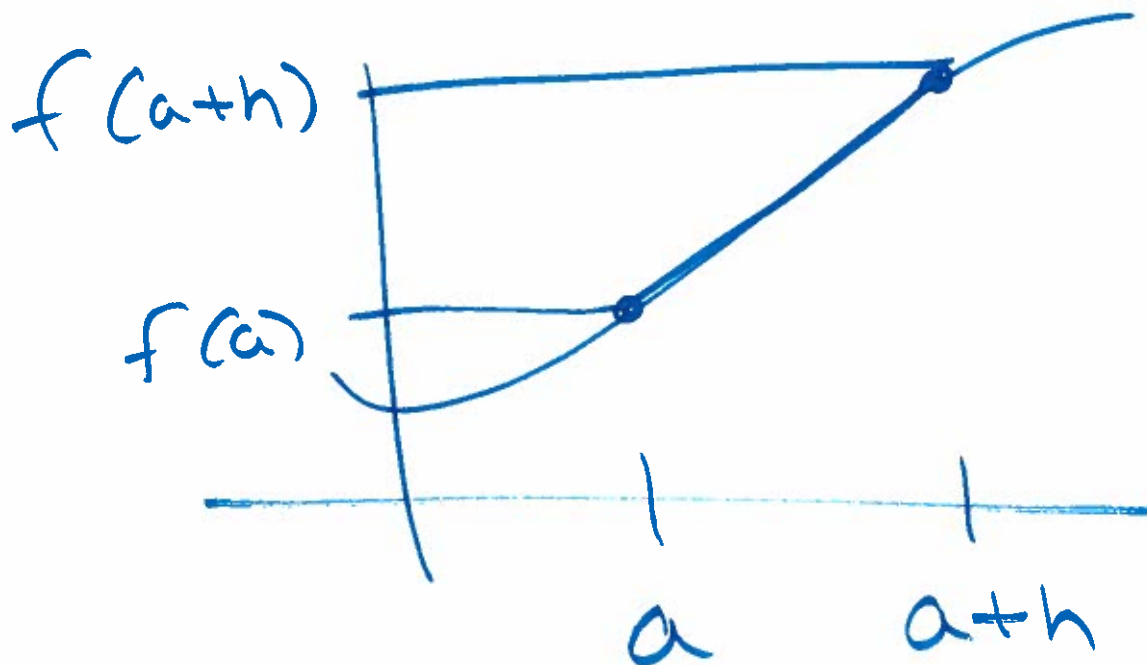
The weight will be within .1
of 200

$$\begin{array}{r} 73.014 \\ - 73 \\ \hline .014 \end{array}$$

$$\begin{array}{r} 73 \\ - 72.995 \\ \hline .005 \\ \delta = .005 \end{array}$$

As long as the height is
within .005 of 73

Instantaneous Rate of Change



$$m_{\text{sec}} = \frac{\Delta f}{\Delta x} = \frac{f(a+h) - f(a)}{h}$$

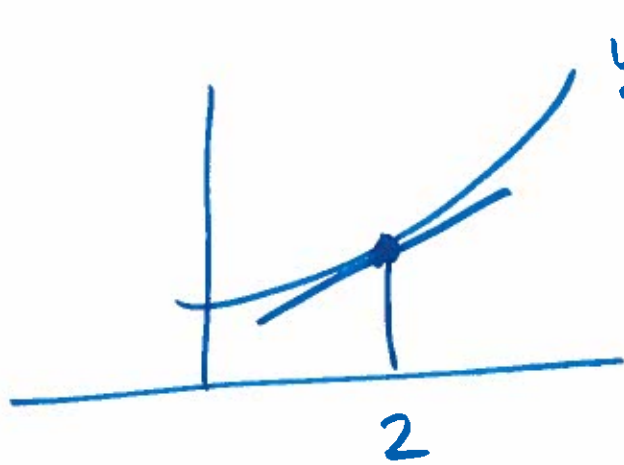
Definition of Derivative

= Instantaneous rate of change

= Slope of Tangent Line

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Ex



$$m_{\text{tan}} = 4$$

$$P \in (2, 5)$$

Slope of Tangent line at $x = 2$

$$f(x) = x^2 + 1$$

$$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = m_{\text{tan}}$$

$$f(2+h) = (2+h)^2 + 1$$

(2+h)(2+h)
FOIL.

$$= 4 + 4h + h^2 + 1$$

$$f(2) = 4 + 1$$

$$f(2+h) - f(2) = 4h + h^2$$

$$\frac{f(2+h) - f(2)}{h} = \frac{4h + h^2}{h} = \frac{h(4+h)}{h}$$

$$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{4+h}{h} = 4$$

Finding

Tan at
a point "a"

Tan at
any point "x"

Tan = derivative

$$\frac{dy}{dx}$$

$$D_x$$

$$\frac{d}{dx}$$

$$f(x)$$

$$f'(x)$$

$$\frac{dy}{dx}$$

$$\Big|_{x=a}$$

$$D_x(a)$$

$$\frac{d}{dx} f(a)$$

$$f'(a)$$

$$f'(x) =$$

$$\lim_{h \rightarrow 0}$$

$$\frac{f(x+h) - f(x)}{h}$$

Use definition of derivative to

find $f'(x)$ if $f(x) = x^2 - 12x + 3$

$$f(x+h) = (x+h)^2 - 12(x+h) + 3$$

$$= x^2 + 2xh + h^2 - 12x - 12h + 3$$

$$f(x) = x^2 - 12x + 3$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - 12h}{h}$$

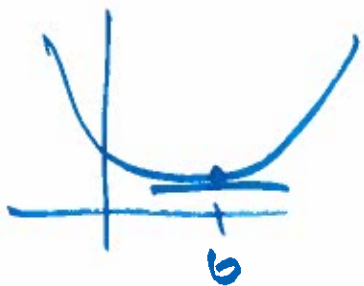
$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} (2x + h - 12)$$

$$\lim_{h \rightarrow 0} 2x + h - 12 \quad f'(x) = 2x - 12$$

$$f'(2) = 2 \cdot 2 - 12 = -8$$

$$f'(5) = 10 - 12 = -2$$

$$f'(6) = 12 - 12 = 0$$



Linear $Y = ax + b$
gives.

$$Y_1 = a$$

Quadratic $Y = ax^2 + bx + c$
gives.

$$Y_2 = 2ax + b$$

Cubic $Y = ax^3 + bx^2 + cx + d$

$$Y_3 = 3ax^2 + 2bx + c$$

Quartre $Y = ax^4 + bx^3 + cx^2 + \dots$

$$Y_4 = 4ax^3 + 3bx^2 + 2cx + d$$

GROUP NAME: <u>WE HAVE BUSINESS</u> Date: <u>11/2/14</u>	Student Names (First and Last) Speaker/Presenter: <u>Yasmin Silverio</u> Writer/Prep: <u>Simar Kalra</u>
Independant Variable (x-axis): <u>Year</u> Dependant Variable (y-axis): <u>Interest Rate of CD</u>	Leader/Collaborator: _____

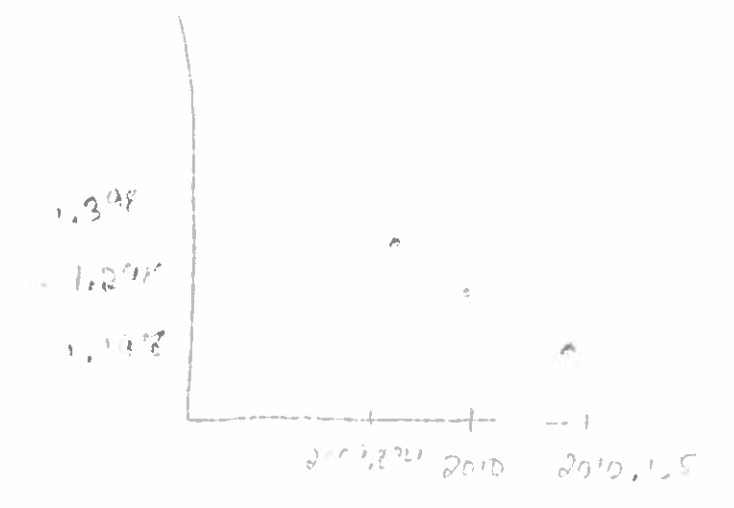
Conclusion (in words): As long as we are in .1257 of 2010, the interest rates would be between .1 of 1.298.

Supporting Work:

$A = 2010$ $L = 1.298$ $E = 0.10$	$\delta = 2010 - 2009.8743$ $\delta = 0.1257$	$\delta = 2010 - 2010.135$ $\delta = -0.135$
---	--	---

Intersection

$y_1 \text{ \& } y_3$	$x = 2009.8743$	$y = 1.398$
$y_2 \text{ \& } y_4$	$x = 2010.135$	$y = 1.198$
$y_3 =$	$1.298 + .1$	
$y_4 =$	$1.298 - .1$	



GROUP NAME: <u>J Illuminati</u>	Student Names (First and Last)
Date: <u>2/12</u>	Speaker/Presenter: <u>Ryan Piotrowski</u>
Independent Variable (x-axis): <u>year</u>	Writer/Prep: <u>Jason Chen</u>
Dependant Variable (y-axis): <u>price</u>	Leader/Collaborator: <u>Bishop Beer</u>

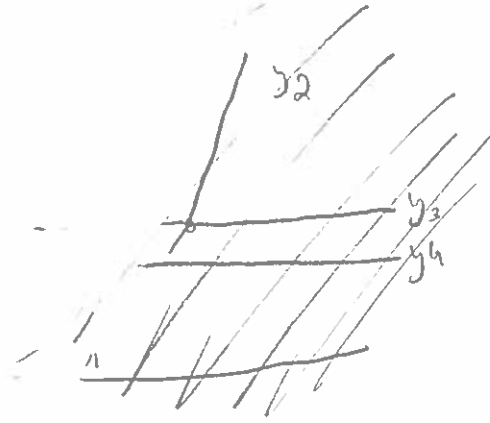
Conclusion (in words): as long as the year is 2004.64 & 2005.68 the price is within 2.005 and 2.2059

Supporting Work:

y_1, y_4 $x = 104.36$	$y = 2.005$	y_2, y_3 $x = 105.68$	$y = 2.2059$
----------------------------	-------------	----------------------------	--------------

y_1, y_3 $\lim \rightarrow 2.1059$
 $x = 103.8609$ $y = 2.2059$

$\lim = 2.1059$



$$y_3 = 2.1059 + .11$$

$$y_4 = 2.1059 - .1$$

y_1, y_4

y_2, y_3

2002 $x = 102.53$ $y = 2.0059$

2005 $x = 105.68$ $y = 2.2059$

2002

2005

GROUP NAME: Money makers

Date: 2/12/13

Student Names (First and Last)

Speaker/Presenter: Bryan

Independent Variable (x-axis): Year

Writer/Prep: Edna

Dependant Variable (y-axis): Crime rate in Detroit

Leader/Collaborator: Monica

Conclusion (in words):

Between the years 2010.84 and 2011.85 the crime rate will be between .528 and .548

Supporting Work:

$$y^1 = \text{Quad Reg 1} (x \leq 2011) + 0.02$$

$$y^2 = \text{Quad Reg 2} (x \geq 2011)$$

$$y_3 = .538 + .01$$

$$y_4 = .538 - .01$$

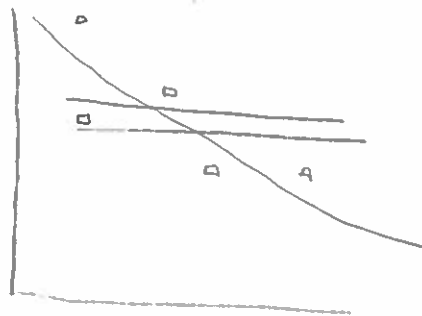
$$y_1 \approx y_2$$

Intersection

$$x = 2011 \quad y = .538$$

$$y_1 \approx y_3$$

$$x = 2010.84 \quad y = .548$$



GROUP NAME: TI rates

Date: 02/12/2013

Student Names (First and Last)

Speaker/Presenter: Shanon Isoe

Independent Variable (x-axis): Bandwidth

Writer/Prep: Onur Turkan / Rasou Pate

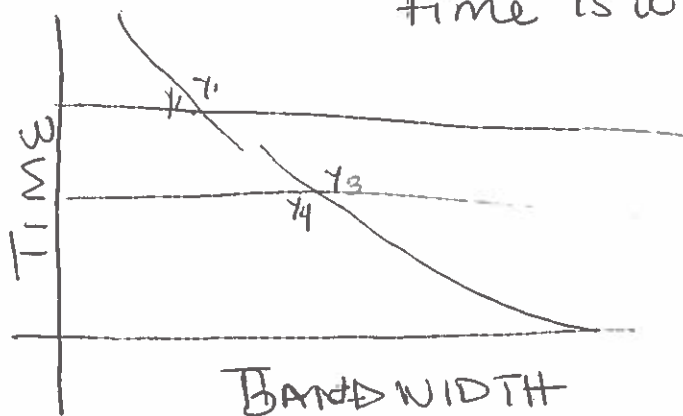
Dependent Variable (y-axis): Time

Leader/Collaborator: George Cumberland

Conclusion (in words): As long as your bandwidth ~~speed~~ uph remains between 78.20uph - 82.342uph your download time will be between 40.121min - 42.121min

Supporting Work:

As long as the time is within $\Delta \rightarrow 2 \text{ min}$.



$$\begin{aligned}
 & \rightarrow 82.342 - 80 \\
 & = 2.342 \\
 & 78.20 - 80 \\
 & = -1.8 \\
 a & = 80 \\
 \text{Limit} & = 40.121 \text{ min} \\
 \epsilon_0 & = 1
 \end{aligned}$$

x	L_2
40	60
60	50
80	40
120	20
160	10

$$\begin{aligned}
 & x_2 = 80.342 \dots & y_2 & 40.121 \text{ min} \\
 & x_3 = 78.2053 \dots & y_3 & 42.121 \text{ min}
 \end{aligned}$$

$$\Delta = -1.8$$

GROUP NAME: <u>BEST FRIENDS</u>	Student Names (First and Last)
Date: <u>2/12/2014</u>	Speaker/Presenter: <u>Vincent Adams</u>
Independant Variable (x-axis): <u>YEARS</u>	Writer/Prep: <u>LAUREN DORO</u>
Dependant Variable (y-axis): <u>CAR SALES</u>	Leader/Collaborator: <u>Elliot Bner</u>

Conclusion (in words):
 AS LONG AS THE YEAR IS BETWEEN YEAR 2010.9999 AND 2011.0001, THE SALES WILL BE WITHIN 10 CARS OF 323602 CARS.

Supporting Work:

YEAR	CARS
X	Y
2009	290292
2010	274555
2011	284087
2012	487480
2013	591192

LTN
 $y = ax + b$
 $a = 81672.5$
 $b = -163257680.9$

QUD
 $y = ax^2 + bx + c$
 $a = 31057.5$
 $b = -124851592.5$
 $c = 1.254367E11$

$L = 323602$
 $E = 10$

$$y_1 = (y_Q)(x \leq a)$$

$$y_2 = (y_L)(x \geq a)$$

$$y_3 = L + E$$

$$= (323602) + (10)$$

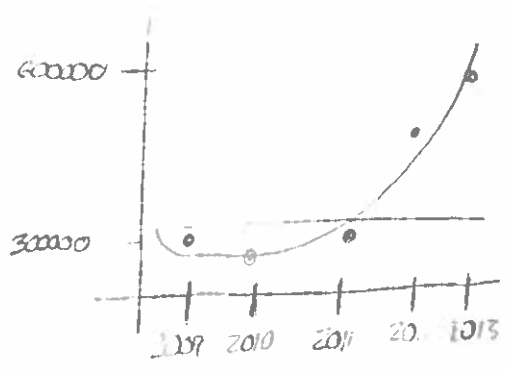
$$y_4 = L - E$$

$$= (323602) - (10)$$

INT.	X	Y
$\rightarrow y_1 + y_4$	2010.9999	323592
$\rightarrow y_2 + y_3$	2011.0001	323612
$\rightarrow y_2 + y_1$	2011.0001	323612

$$\Delta = |2011.0001 - 2011.0000| = 0.0001$$

$$\Delta = |2010.9999 - 2011| = 0.0001$$



GROUP NAME:

Student Names (First and Last)

Date: 2/12/2014

Speaker/Presenter: _____

Independent Variable (x-axis): time (hours)

Writer/Prep: Karol Zarski

Dependant Variable (y-axis): disk usage (mB)

Leader/Collaborator: Nader Shenouda

Conclusion (in words):

As long as the time the computer is used between 1.999 and 2.0002 hours, then the disk usage is between 2500.1 mB and 2499.9 mB.

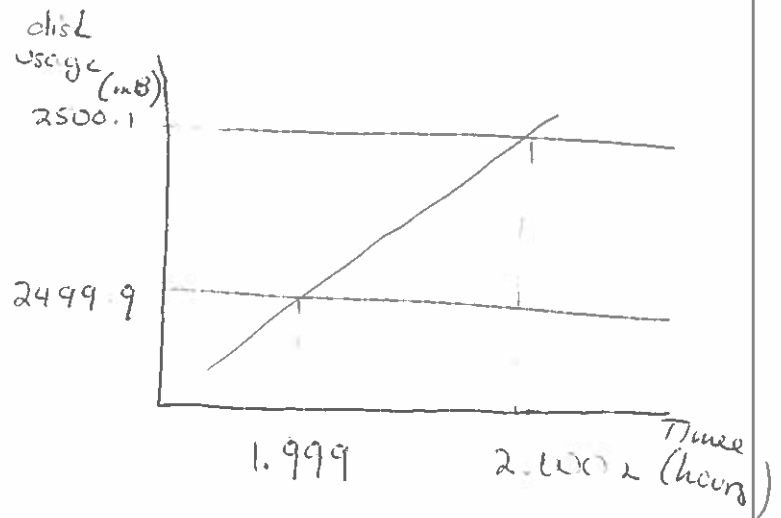
Supporting Work:

$$Y_1 = \text{Quart Reg} / (x \leq a)$$

$$Y_2 = \text{Quad Reg} / (x \geq a) + 308.60$$

$$Y_3 = 2500 + .1$$

$$Y_4 = 2500 - .1$$



~~1st corner~~

Y_1

Y_4

intersection

$$x = 1.999 \quad y = 2499.9$$

$$\epsilon = .1$$

$$\delta = .0002$$

Y_2

Y_3

intersection

$$x = 2.0002 \quad y = 2500.1$$

GROUP NAME: Polarz Bearz
 Date: _____
 Independant Variable (x-axis): Year
 Dependant Variable (y-axis): Deaths

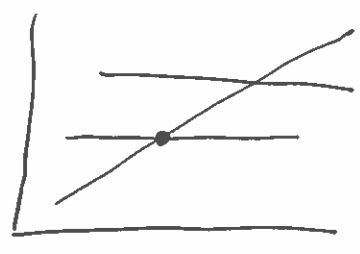
Student Names (First and Last) Trey Murnil
 Speaker/Presenter: SFA Kausalya Mannum
 Writer/Prep: Sheila Mae Wan
 Leader/Collaborator: Frewinot Bekele

Conclusion (in words):
~~Between 2010 and 2011~~
 As long as the ~~deaths are~~ ^{the year} ~~are~~ ^{0.769} Between 2010^{0.769} and 2011 the
 the deaths are between 2.41 and 2.43 million.

Supporting Work:
 $y_1 = \text{Reg Exp} / (x \leq 2011) + .0095$
 $y_2 = \text{Reg Lin} / (x \geq 2011)$
 $y_3 = 2.42 + .01$
 $y_4 = 2.42 - .01$

$L = 2.42$
 $a = 2011$

a) 1st curve
 y_1 2nd ~~curve~~
 y_4 curve intersect
 intersection
 $X = 2010.9241$ $Y = 2.41$



b) 1st curve
 y_2 Intersection
 y_3
 $X = 2011.0769$ $Y = 2.43$
 $d = .8472$

