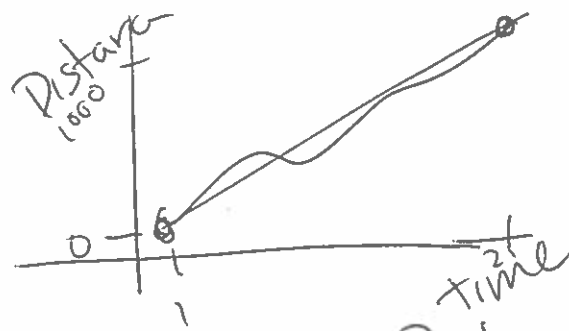


# Study of Change

Functions  $\left\{ \begin{array}{l} \text{Data} \\ \text{Graphs} \\ \text{Equations} \end{array} \right.$

## Averaged Rate of Change

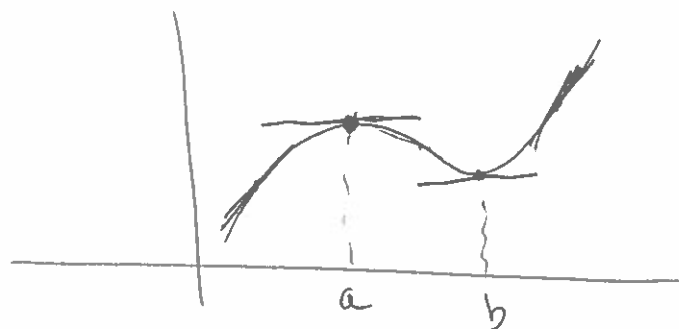
2 data points = slope of line



$$\frac{\Delta y}{\Delta x} = \frac{1000 - 0 \text{ mi}}{2 - 1 \text{ hr}} = 50 \text{ mph.}$$

## Instantaneous Rate of Change

1 data point = slope of Tangent Line



at "a"  
at "b" } travelling

# Limits

Invented 2 points  $\rightarrow$  1 point.

$$\lim_{h \rightarrow 0} f(h) = f(0)$$

Three ways TO Find.

1. Graph

2. Points  $\rightarrow$

3. Algebra

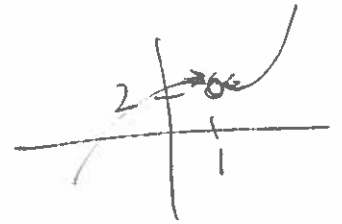
EZ: Plug in

med:

Factor

Hard

$$\lim_{x \rightarrow 1} f(x) = 2$$



$$1 \quad | \quad 2 \quad | \quad 3 \quad | \quad 4 \quad | \quad 5 \quad | \quad 6$$

$$\lim_{x \rightarrow 1} \frac{x+1}{x+1} = \frac{1+1}{1+1} = 1$$

$$\lim_{x \rightarrow 1} \frac{x^2-1}{x-1} = \frac{0}{0}$$

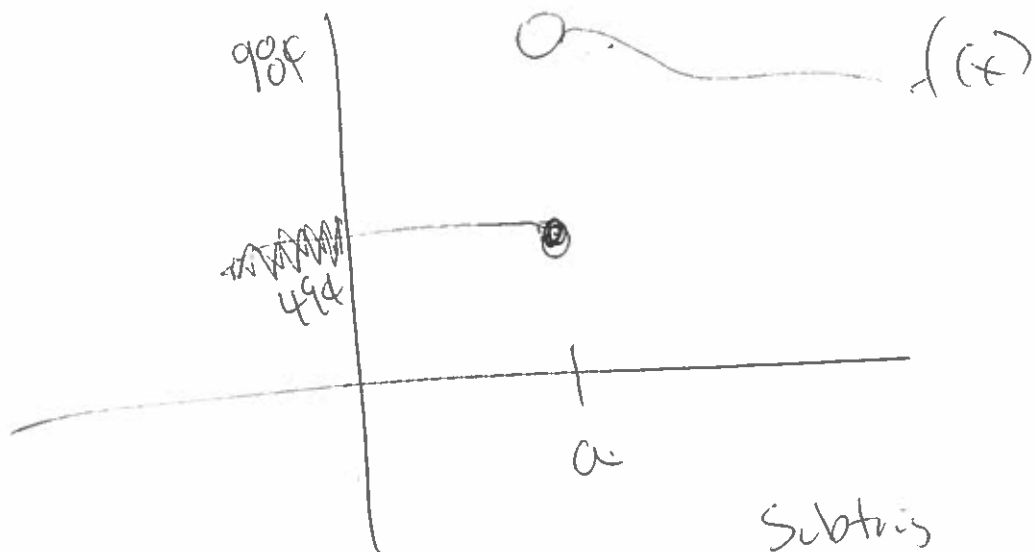
$$\frac{(x-1)(x+1)}{(x-1)}$$

$$\lim_{x \rightarrow 1} x+1 = 2$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{x-4} - 1}{x-1} =$$

# - Two Sided Limits

$$\lim_{x \rightarrow a^-} f(x) \quad \text{or} \quad \lim_{x \rightarrow a^+} f(x)$$



Adding  
to letter to  
get to  
"a"

Then you  
Pay 49¢

Subtracting  
from the letter  
to get to  
102.

Then you  
Pay 99¢

# Put Data In

**STAT** 1: Edit

Enter Data

L1	L2
7	26
2	26
5	16
21	:
5	0

**Y=**  $\Delta$  **ENTER** **ZOOM** 9:



**STAT**  $\rightarrow$  Calc 0: ExpReg

$$Y = 26.4 \cdot X^{.74...^x}$$

**Y=** **VARs** 5:  $\rightarrow$   $\rightarrow$  1:



$$Y_1 = \text{Expreg} / (x \leq 3)$$

STAT  $\rightarrow$  calc 4: lin reg

$$Y_2 =$$

$\sqrt{=}$   $\checkmark$  VARS 5.  $\rightarrow$   $\rightarrow$  1.

$$Y_2 = \text{lin reg} / (x \geq 3)$$

2nd WINDO  $\checkmark$   $\checkmark$   $\rightarrow$  ASK ENTER  
1st GRAPH

$$\lim_{x \rightarrow 3^-} R_{\text{exp}} \cong 11.013$$

$$\lim_{x \rightarrow 3^+} R_{\text{lim}} = 12$$

Right End Behavior

$$\lim_{x \rightarrow \infty} \text{RegLn}(x) = -\infty$$

$$x \rightarrow \infty$$

Left End Behavior

$$\lim_{x \rightarrow -\infty} \text{RegExp}(x) = \infty$$

$$x \rightarrow -\infty$$

$$\lim_{x \rightarrow 3^-} \text{regexp}(x) = 11.013$$

$$x \rightarrow 3^-$$

$$\lim_{x \rightarrow 3^+} \text{regln}(x) = 12$$

$$x \rightarrow 3^+$$

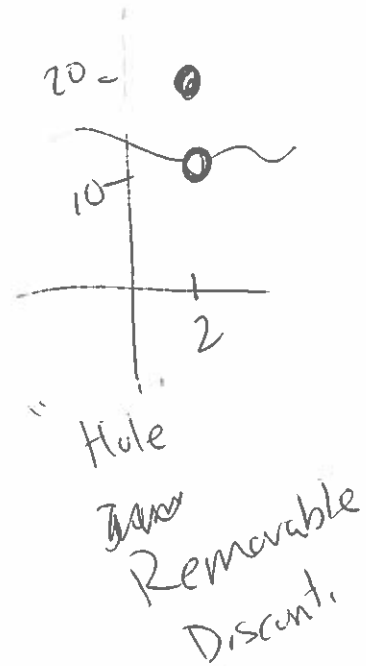
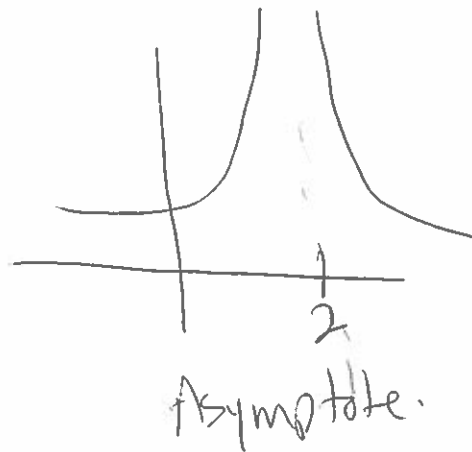
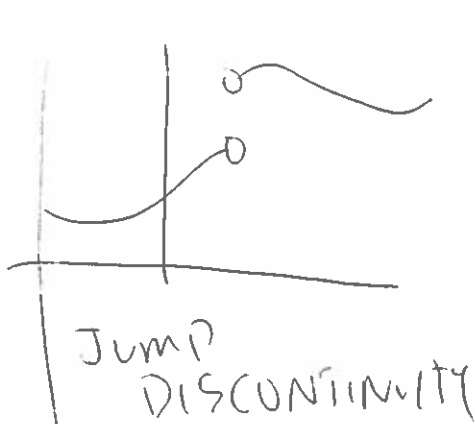
$$\lim_{x \rightarrow -\infty} \text{regexp}(x) = \infty$$

$$x \rightarrow -\infty$$

$$\lim_{x \rightarrow \infty} \text{regln}(x) = -\infty$$

$$x \rightarrow \infty$$

# CONTINUITY



Polynomials Continuous  
Everywhere

## Rules

1. Limit exists  $\lim_{x \rightarrow a} f(x)$

2.  $f(a)$  exist.

3.  $\lim_{x \rightarrow a} f(x) = f(a)$

$$y_1 = \dots + .02$$

$$y_2 = \dots$$

$$\lim_{x \rightarrow 2011} f(x) = .530$$

$$x \rightarrow 2011$$



GROUP NAME: BEST FRIENDS

Student Names (First and Last)

Date: 2/10/2014

Speaker/Presenter: VANIE L...

Independent Variable (x-axis): YEAR

Writer/Prep: LAUREN D...

Dependant Variable (y-axis): SALES

Leader/Collaborator: ELLIOT BAER

Conclusion (in words):

LOOKING FORWARD, WE THOUGHT THE ELECTRIC CAR SALES WOULD BE 323,602, BUT LOOKING BACK, IT WAS 385,717.

Supporting Work:

x	y
2009	290292
2010	274555
2011	284064
2012	487480
2013	592192

QUAD

$$y = ax^2 + bx + c$$

$$a = 31057.5$$

$$b = -124531592.5$$

$$c = 12543637211$$

$$y = ax^2 + bx + c / (x \leq 2011)$$

$$y_2(2011) = 323602$$

LIN

$$y = ax + b$$

$$a = 81677.5$$

$$b = -1638376809$$

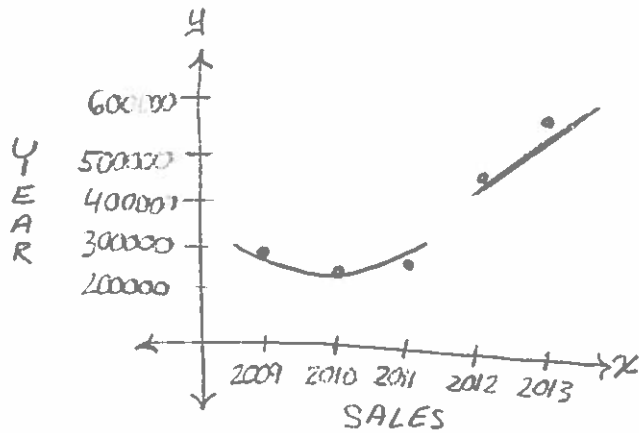
$$y = b + ax / (x \geq 2011)$$

$$y_1(2011) = 385717$$

$$y_{1b} = (b + ax / (x \geq 2011)) - 62115$$

ALTERED

	QUAD	LIN
2011	323602	323602



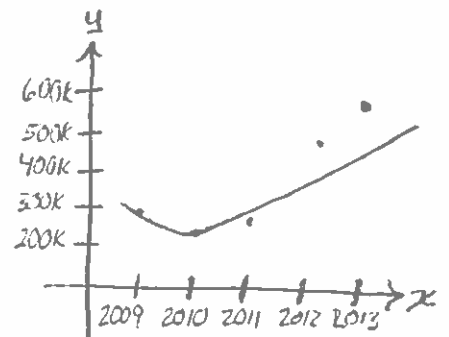
$$\lim_{x \rightarrow 2011^+} (\text{QUAD}) = 323602$$

$$\lim_{x \rightarrow 2011^-} (\text{LIN}) = 385717$$

$$\lim_{x \rightarrow \infty} (\text{QUAD}) = \infty$$

$$\lim_{x \rightarrow \infty} (\text{LIN}) = \infty$$

limit limit  
323602



ALTERED

GROUP NAME: Cha-Ching

Date: 02/10/14

Student Names (First and Last)

Speaker/Presenter: Trey Murrill

Writer/Prep: Sheila Mae Gian

Leader/Collaborator: Tatiana C.

Independent Variable (x-axis): year

Dependant Variable (y-axis): revenue

Conclusion (in words):

ALL THE YEARS BEFORE 2011 WE WERE MAKING 23.17 mil AND  
 ALL THE YEARS AFTER 2011 WE'RE ~~MAKING 24.2 mil~~  
 EXPECTING TO MAKE  $\infty$

Supporting Work:

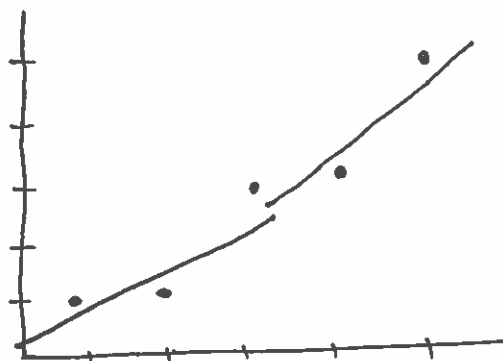
$$Y_1 = \text{RegExp} / (x \leq 11)$$

$$Y_2 = \text{RegLin} / (x \geq 11)$$

DATA

L1	L2
13	35
12	27
11	26
10	17
9	16

**STAT** 1: Edit



$$Y_1 = 11^{+ \text{RegExp}} = 23.17$$

$$Y_2 = 11^{- \text{RegLin}} = 24.2$$

1.  $\lim_{x \rightarrow 11^+} \text{RegExp}(x) = 23.17$

2.  $\lim_{x \rightarrow 11^-} \text{RegLin}(x) = 24.2$

3.  $\lim_{x \rightarrow -\infty} \text{RegExp}(x) = -\infty$

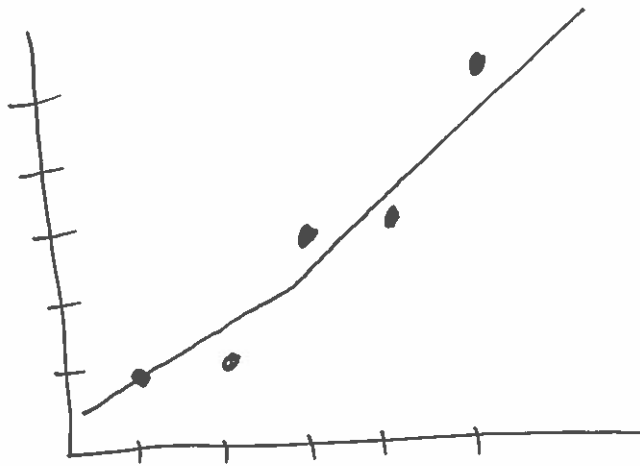
4.  $\lim_{x \rightarrow \infty} \text{RegLin}(x) = \infty$

GROUP NAME: Cha - Ching

Student Names (First and Last)

Date: 02/10/14Speaker/Presenter: Trey MurrillIndependent Variable (x-axis): YearWriter/Prep: Sheila Mae GanDependant Variable (y-axis): revenueLeader/Collaborator: Tatiana C.Conclusion (in words): 2011, revenue goes up 1.026

Supporting Work:



$$Y_1 = 11^{+} R_{exp} = 23.17$$

$$Y_2 = 11^{-} R_{lin} = 24.2$$

Difference is 1.026

GROUP NAME: Functional Paradigm

Date: 2/10/14

Student Names (First and Last)

Speaker/Presenter: \_\_\_\_\_

Independent Variable (x-axis): disk usage in mB

Dependant Variable (y-axis): time in hours

Writer/Prep: Karol Zurski

Leader/Collaborator: Nader Shenarfa

Conclusion (in words):

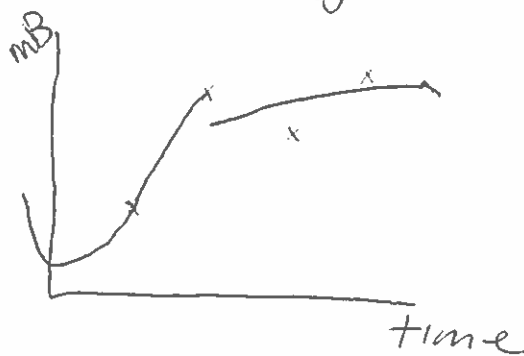
As we approach 2 hours, we expect to be using 2500 mB ~~per hour~~. As we keep using the computer we expect the disk to crash ( $-\infty$ ) because we used a quadratic regression.

Supporting Work:

x	y
0	1000
1	1500
2	2500
3	2250
4	2600

$$y_1 = \text{Quart Reg} / (x \leq 2)$$

$$y_2 = \text{Quad Reg} / (x \geq 2)$$



$$\lim_{x \rightarrow 2^-} f(2) = \cancel{2500} \quad \text{Quart Reg} \left(\frac{x}{2}\right) = 2500$$

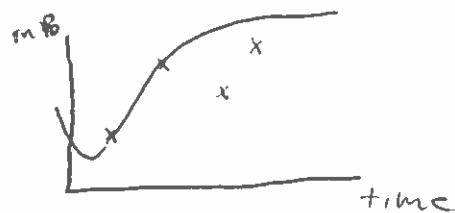
$$\lim_{x \rightarrow 2^+} f(2) = \cancel{2191.4} \quad \text{Quad Reg}(x) = 2191.4$$

Project 3

$$\lim_{x \rightarrow -\infty} \text{Quart Reg}(x) = \infty$$

$$\lim_{x \rightarrow \infty} \text{Quad Reg}(x) = -\infty$$

$$y_2 = \text{Quad Reg} / (x \geq 2) + 308$$



$$\lim_{x \rightarrow 2} \text{Quad Reg}(x) = 2530$$

GROUP NAME: Illuminati  
 Date: 2/10  
 Independent Variable (x-axis): year  
 Dependant Variable (y-axis): pieces

Student Names (First and Last)  
 Speaker/Presenter: Ryan Piotrowski  
 Writer/Prep: Danyan Zhou  
 Leader/Collaborator: \_\_\_\_\_

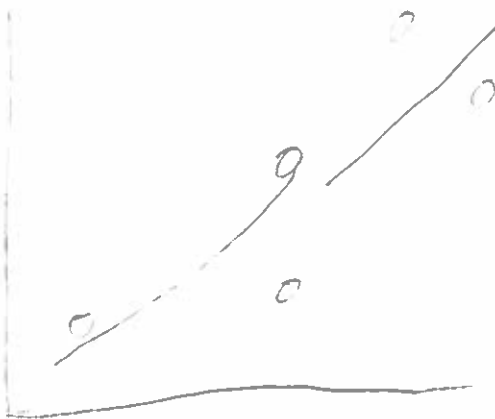
Conclusion (in words): As we approach 2005

Supporting Work:

$Y_1 =$  Linear regression /  $(x \leq 105)$

$Y_2 =$  Exponential reg /  $(x \geq 105)$

year	#
95	1
102	1.54
105	2.5
112	3.99
114	3.3



$x$	$Y_1$	$Y_2$
105	2.1059	2.1016

GROUP NAME:

Student Names (First and Last)

Date: 2/10

Speaker/Presenter: Ryan Piotrowski

Independent Variable (x-axis): \_\_\_\_\_

Writer/Prep: Danyan Zhou

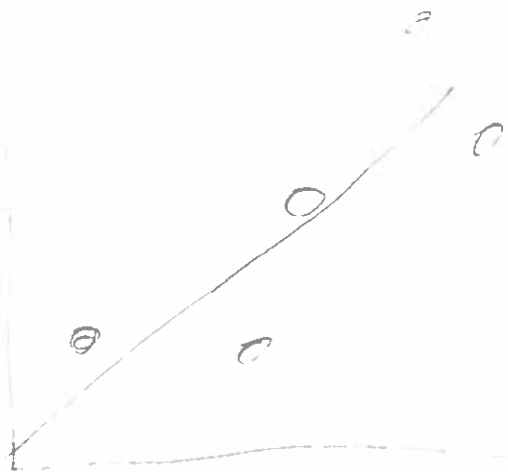
Dependant Variable (y-axis): \_\_\_\_\_

Leader/Collaborator: \_\_\_\_\_

Conclusion (in words):

Supporting Work:

$$\lim_{x \rightarrow 105} f(x) \rightarrow -0.27$$



	$y_1$	$y_2$
105	2.1059	2.1016

GROUP NAME: Money Makers

Student Names (First and Last)

Date: 2/10/14

Speaker/Presenter: Bryan G

Independent Variable (x-axis): year

Writer/Prep: Echa Onyruke

Dependant Variable (y-axis): crime rate

Leader/Collaborator: MONICA KONCHANDRAN

Conclusion (in words):

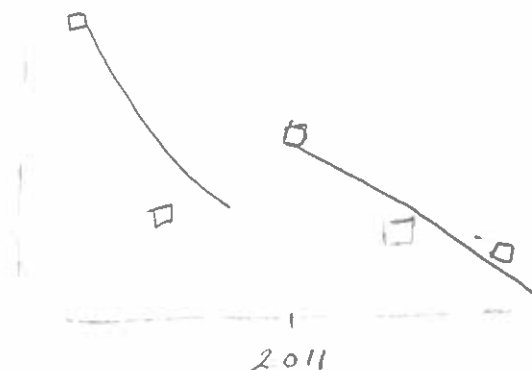
As we approach 2011 the rate is expected to be 0.538  
or 54%

Supporting Work:

x	y
2009	.75
2010	.52
2011	.59
2012	.44
2013	.39

$y_1 = \text{Reg Exp} / (x \leq 2011)$

$y_2 = \text{reg lin} / (x \geq 2011)$



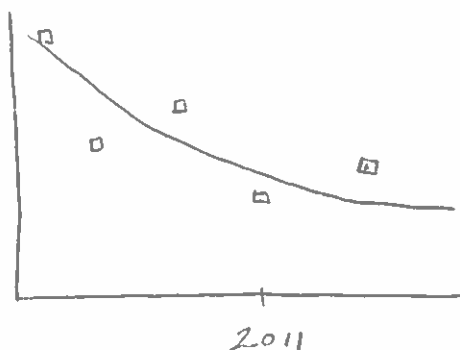
$\lim_{x \rightarrow 2011^-} = 0.518$

$y_1 = .01x^2 + -40.3x + 40602.677 / (x \leq 2011) + 02$

$\lim_{x \rightarrow 2011^+} = 0.538$

$y_2 = -.07x + 161.418 / (x \geq 2011)$

$\lim_{x \rightarrow -\infty} = \infty$



$\lim_{x \rightarrow \infty} = -\infty$

$\lim_{x \rightarrow 2011} = 0.538$  continuous

GROUP NAME: TI rates

Date: 02/10/2014

Student Names (First and Last)

Speaker/Presenter: Shanon Isoe

Independent Variable (x-axis): bandwidth

Writer/Prep: Onur Turkay

Dependent Variable (y-axis): time

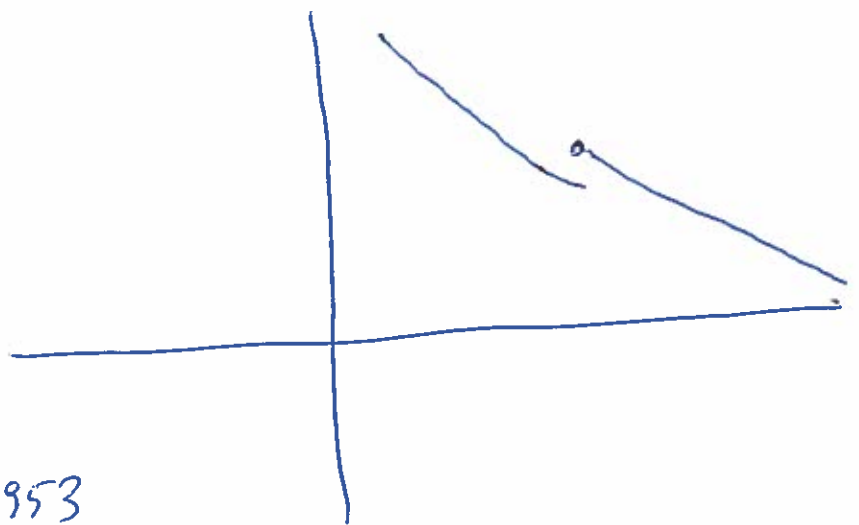
Leader/Collaborator: Fulvia

Conclusion (in words): - the av. time limit with  $\lim_{x \rightarrow 80^-}$  is 35.953 min  
 $\lim_{x \rightarrow 80^+}$  is ~~40~~ 41.121 min

Supporting Work:

GB	Time
40	60
60	50
80	40
120	20
160	10

$y_1 = \text{exp reg } (x \leq 80)$   
 $y_2 = \text{lin reg } (x \geq 80)$



$\lim_{x \rightarrow 80^-} \text{reg exp} = 35.953$

$\lim_{x \rightarrow 80^+} \text{reg lin}(x) = 41.121$

$\lim_{x \rightarrow -\infty} \text{reg exp}(x) = +\infty$

$\lim_{x \rightarrow \infty} \text{reg ln}(x) = -\infty$

- Continuous

Limit exists

$\lim_{x \rightarrow a} f(x) = 5.168$



GROUP NAME: TI rates

Date: 02/10/2014

Student Names (First and Last)

Speaker/Presenter: Sharon Isoe

Independent Variable (x-axis): bandwidth

Writer/Prep: Onur Turkan

Dependant Variable (y-axis): time

Leader/Collaborator: Purav Patel

Conclusion (in words): - limit only exists continuous get speed of 5.168 min.

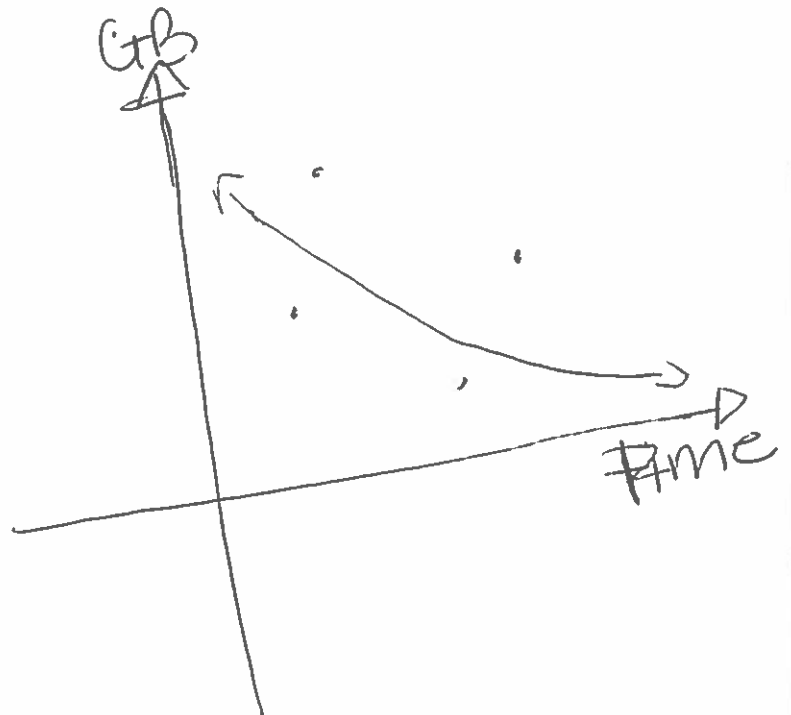
Supporting Work:

$$y = \exp \text{ Reg} / (x \leq 80) + 5.168$$

x	y1	y2
80	41.121	35.953

$$\begin{array}{r} 41.121 \\ - 35.953 \\ \hline 5.168 \end{array}$$

$$\text{Exp.} = + 5.168$$



speed of 41.121 min

GROUP NAME: <u>Polarz Bearz</u>	Student Names (First and Last)
Date: <u>2/10/14</u>	Speaker/Presenter: <u>Natalie Casillo</u>
Independent Variable (x-axis): <u>year</u>	Writer/Prep: <u>Kanisha Mamm...</u>
Dependant Variable (y-axis): <u>deaths due to AIDS</u>	Leader/Collaborator: <u>Fremont Polk...</u>

Conclusion (in words):

As we increase the year to 2011, the number of deaths due to AIDS is 2.7 million. As we go back in time there will be no deaths.

Supporting Work:

x	y mil
2009	2.1
2010	2.3
2011	2.6
2012	2.4
2013	2.7



$$f_1 = \exp(x) / x \leq 1$$

$$f_2 = \ln(x) / x \geq 2$$

not very  
good .0095

$$f(x) = .0095$$

$x \rightarrow 2011$

lim exp - a 2nd  
 $x \rightarrow 20$   
 $x \rightarrow \infty$   
 number  
 $x \rightarrow \infty$   
 fraction  
 $\rightarrow \infty$  = 3.57

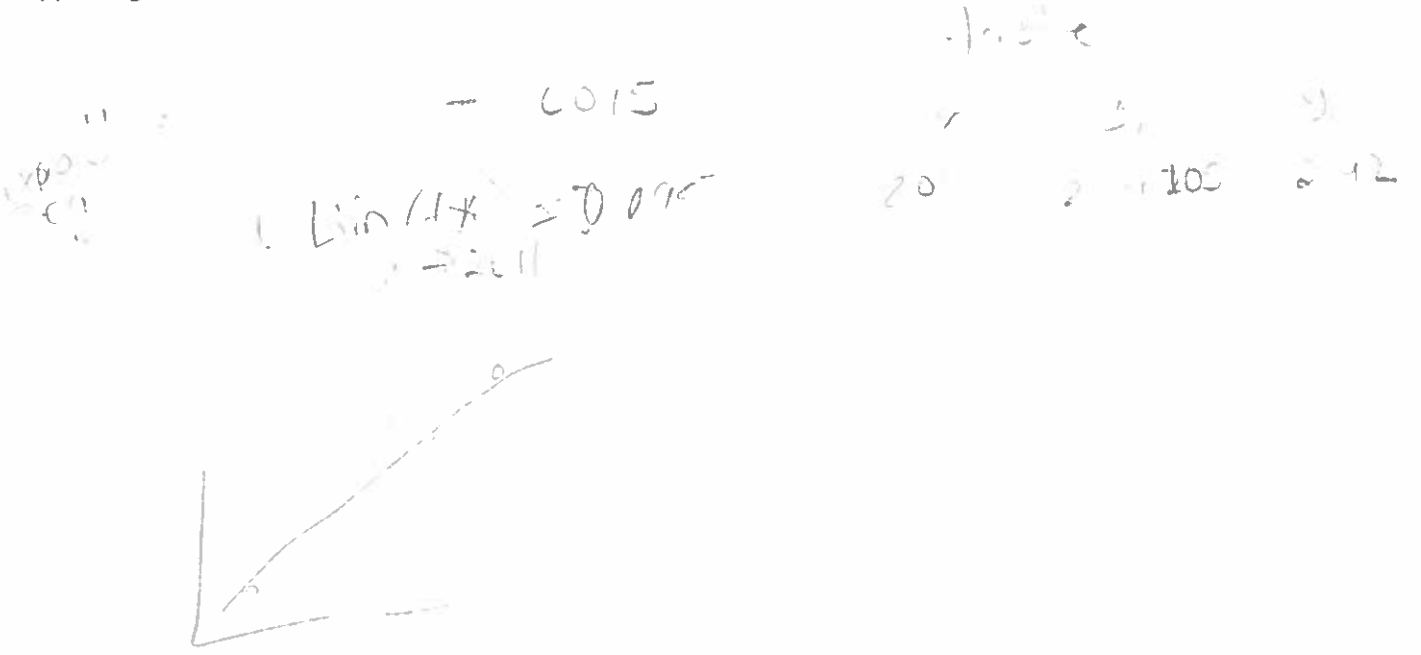


GROUP NAME:	PD 102 Beatz	Student Names (First and Last)
Date:	2/1/14	Speaker/Presenter: <u>Naiya &amp; Gauri</u>
Independent Variable (x-axis):	year	Writer/Prep: <u>Kausiya Mahesh</u>
Dependant Variable (y-axis):	leaths	Leader/Collaborator: <u>Arewa Betele</u>

Conclusion (in words):

AS we approach 2011 there is 0.95 leaths due to HIV

Supporting Work:

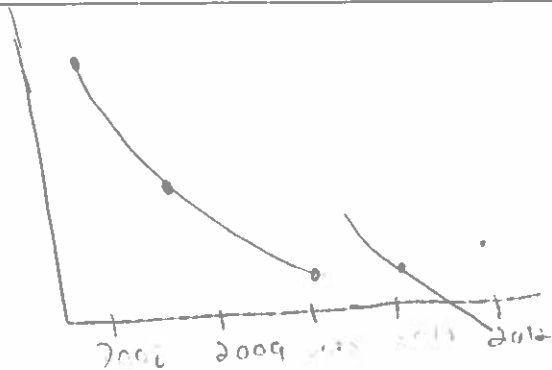


GROUP NAME: <u>We Make Business</u> Date: <u>2/10/14</u>	Student Names (First and Last) Speaker/Presenter: <u>Christina Trujillo</u> Writer/Prep: <u>Yaimn Silverio</u> Leader/Collaborator: <u>Simar Kalra</u>
Independant Variable (x-axis): <u>Years</u> Dependant Variable (y-axis): <u>Interest Rate</u>	

Conclusion (in words):

Supporting Work:

X	Y
2008	3.71
2009	1.53
2010	.29
2011	.38
2012	.58



$$y_1 = \text{RegQuad} / (x \leq 2010) \quad y_1 = .4135x^2 + -1749.4411x + 1756933.338 / (x \leq 2010)$$

$$y_2 = \text{RegLin} / (x \leq 2010) \quad y_2 = .465x + -174.614x + 1756933.338 / (x \leq 2010)$$

x	x1	y2
2010	.428	1.298

**Conclusion** ↓

Lim  $R_{quad} = .428$   
 $x \rightarrow 2010^-$

The limit as you progress to 2010 by looking at Quad regression gives us .428% interest rate for the CD

Lim  $R_{lin} = 1.298$   
 $x \rightarrow 2010^+$  The limit for linear regression as we approach 2010 from the right the interest rate is 1.298

Lim  $R_{quad} = \infty$   
 $x \rightarrow \infty$

Lim  $R_{lin} = \infty$  In a billion. This is the interest rate as  $x \rightarrow \infty$

Continuous Limit

1/24 page

GROUP NAME: 117 MATH BEINGS

Date: 2/10/14

Student Names (First and Last)

Speaker/Presenter: Christina Rojas

Writer/Prep: Yulian Silverio

Leader/Collaborator: Simon Kalra

Independent Variable (x-axis): years

Dependant Variable (y-axis): Interest Rate

Conclusion (in words): Added .87 to continuous

limit  $\lim_{x \rightarrow 2010} \text{Rate} = .128$

Supporting Work:

Continuous Limit

$$\lim_{x \rightarrow 2010} f(x) = 1.298$$

