Fundamental Theorem of Calculus

w/ Substitution

$$\int f(u)\,du = F(u) + C$$

$$\boxed{\int_a^b f(u)\,du} = F(b) - F(a)$$

or

$$F(u)\Big|_a^b$$

Definite

Area under curve

Substitution / Undoing of Chain Rule.

$$\int e^{x^2} x\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int e^{u} \frac{du}{2}$$

$$= \frac{1}{2}\int e^{u} du$$

$$= \frac{1}{2} e^{u} + c$$

ANS: $= \frac{1}{2}(e^{x^2} + c)$

Ex

$$\int \sin(\cos x)\sin(x)\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$-\int \sin(u)\,du$

$\downarrow$

$+\cos(u) + C$

Ans. $\cos(\cos x) + C$

Ex

$$\int \frac{\sin x}{\cos x}\,dx = \int \frac{1}{\cos x}\cdot \sin x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-\int \frac{1}{u}\,du$

$-\ln|u| + C$

ANS: $-\ln|\cos x| + C$

Ex

$$\int \frac{\sec(\ln x)\tan(\ln x)}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$\int \sec(u)\tan(u)\,du$$

$$\sec(u) + C$$

Ans: $\sec(\ln x) + C$

Ex

$$\int x\sqrt[3]{x-5}\,dx$$

$u = x - 5$

$du = dx$

$x = u+5$

$$\int (u+5)\sqrt[3]{u}\,du$$

$$\int (u+5)u^{1/3}\,du$$

$$\int u^{4/3} + 5u^{1/3}\,du$$

$$u^{7/3} + 5u^{4/3} + C$$

Ans. $(x-5)^{7/3} + 5(x-5)^{4/3} + C$

Ex $\displaystyle\int \frac{1}{1+4x^2}\,dx$

$$\boxed{\int \frac{1}{1+u^2}\,du = \tan^{-1}(u) + C}$$

$u = 2x$

$u^2 = 4x^2$

$du = 2\,dx$

$\frac{du}{2} = \frac{2\,dx}{2}$

$$\int \frac{1}{1+u^2} \cdot \frac{du}{2}$$

$$\frac{1}{2}\left(\tan^{-1}(u)\right) + C$$

Ans $\frac{1}{2}\tan^{-1}(2x) + C$

$$\int \frac{1}{1+x^2}\,dx \qquad \int \frac{x}{1+x^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int \frac{1}{u}\,\frac{du}{2}$$

$$\frac{1}{2}\ln|u| + C$$

$$\frac{1}{2}\ln|1+x^2| + C$$

$$\int \frac{x^2}{1+x^2}\,dx$$

$$1 - \frac{1}{x^2+1}$$

$$x^2+1 \overline{\big)\, x^2 + 0x + 0}$$
$$-(x^2 \qquad + 1)$$
$$\qquad -1$$

$$= \int 1 - \frac{1}{x^2+1}\,dx$$

$$\int dx - \int \frac{1}{1+x^2}\,dx$$

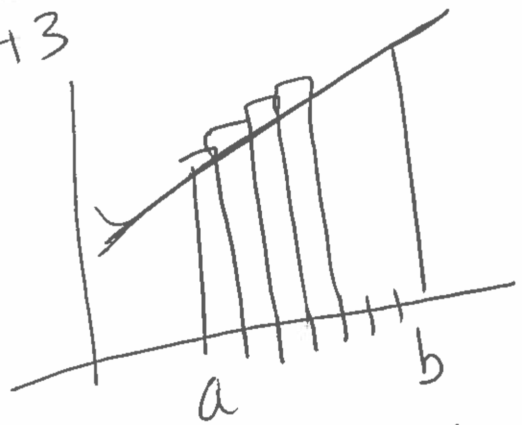
$$x - \tan^{-1}(x) + C$$

#9

	Old x	
Yrs.	Yrs after 2000	X − 2000

$$y = Ax + B$$

#5 p13

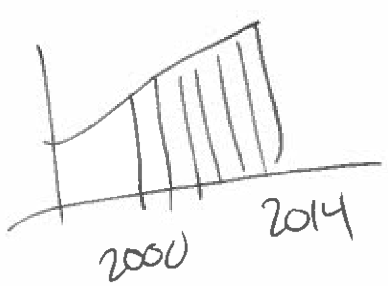


n rectangles

$$\Delta x = \frac{b-a}{n}$$

$$y = 3.2x + 5.5$$

Ex



n = rectangle

$$\Delta x = \frac{2014-2000}{n} = \frac{14}{n}$$

$$\begin{array}{c} A \quad \overset{(a\,+\,\Delta x\, i)}{x} \quad + \quad B \qquad \Delta x \end{array}$$

$$A(n) = \left(\sum_{i=1}^{n} \underset{A}{3.2}\left(\underset{a}{2000} + \underset{\Delta x}{\frac{14}{n}}\, i\right) + \underset{B}{5.5}\right)\underset{\Delta x}{\frac{14}{n}}$$

$$\sum_{i=1}^{n}\left(6400 + \frac{44.8}{n}\, i + 5.5\right)\frac{14}{n}$$

$$\sum_{i=1}^{n} \frac{896\cancel{77}}{n} + \frac{627.2\, i}{n^2}$$

$$\sum_{i=1}^{n}\left(\frac{89677}{n}\right) + \sum_{i=1}^{n} i \cdot \frac{627.2}{n^2}$$

$$\left(\frac{89677}{\cancel{n}}\right) \cdot \cancel{n} + \frac{n \cdot (n+1)}{2} \cdot \frac{627.2}{n^2}$$

$$\sum_{i=1}^{4} 3x = 3x + 3x + 3x + 3x = 3x \cdot 4x = 12x$$

(a) $A(n) = 89677 + \frac{627.2}{2}\left(\frac{n^2+n}{n^2}\right)$

(b) $\lim\limits_{n \to \infty} A(n) = 89677 + 313.6$

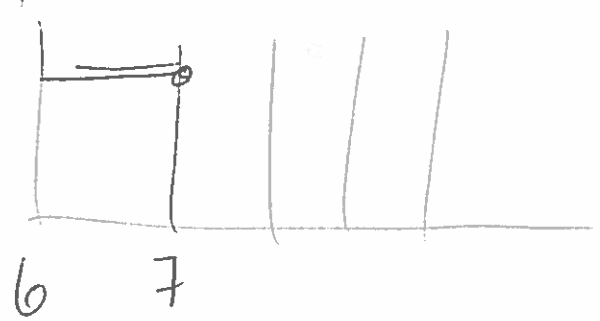
$= 89990.6$

regessi

#4 8T $\text{sum}(\text{seq}(Y_1, X, \underset{\text{Start}}{\cancel{6+\Delta x}\ 7}, \underset{\text{Finsh}}{14}, \underset{\Delta x}{\overset{\Delta x}{1}}) \cdot \frac{\overset{\Delta x}{1}}{\Delta x}$

Right

$6 \xrightarrow{8} 14$



$\text{sum}(\text{seq}(Y_1, X, 6, 13, 1) \cdot 1$

Left