

Rates of Change

ISI m
d12

<u>RATE</u>	<u>VALUE</u>
$\frac{dy}{dx}$	$y(x)$
$\frac{dL}{dy}$	$L(y)$
	ft
\$ per year	miles
miles per hour	years

$$x = \sin y$$

Implicit Differentiation

$$\frac{d}{dx} x = \frac{d}{dx} \sin y$$
$$1 = \cos y \cdot \frac{dy}{dx}$$

$$\frac{d}{dt} x = \frac{d}{dt} \sin t$$

$$\frac{dx}{dt} = \cos t$$

Rates

$$dR/dt = \text{Revenue is change}$$

Values

$$X = \text{price}$$

$$dX/dt = \text{price is change}$$

$$X = \text{Price is } \$1$$

$$dX/dt = \text{Raise the price } \$0.05 / \text{week}$$

$$dR/dt = 22(.05) - 4(\$1)(.05)$$

$$= .9 \text{ \$ / week}$$

$$X = \text{Price} = \$12$$

$$dX/dt = .05 / \text{week?}$$

$$dR/dt = 22(.05) - 4(12)(.05)$$

$$= -1.3$$

Related Rates

Like Implicit differentiation.

except Take $\frac{d}{dt}$ of both side.

t = value = time

Ex

$$V = \frac{4}{3} \pi r^3$$

$$\frac{d}{dt} V = \frac{d}{dt} \frac{4}{3} \pi r^3$$

$$\boxed{\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}}$$

Volume of sphere.

Rates

$$\frac{dV}{dt} = 30 \frac{\text{cm}^3}{\text{sec}}$$

$$\frac{dr}{dt} = ? \frac{\text{cm}}{\text{sec}}$$

Value

$$r = 1 \text{ cm}, 16 \text{ cm}$$

$$160 \text{ cm}^3$$



Balloon

$$r = 1 \quad 30 \frac{\text{cm}^3}{\text{Sec}} = 4\pi (1\text{cm})^2 \frac{dr}{dt} \frac{\text{cm}}{\text{Sec}}$$


$$\frac{30}{4\pi (1)^2} = \frac{dr}{dt} = 2.38 \dots \text{cm/Sec}$$

$$r = 10$$

$$\frac{30}{4\pi (10)^2} = \frac{dr}{dt} = 0.0238 \dots \text{cm/Sec}$$

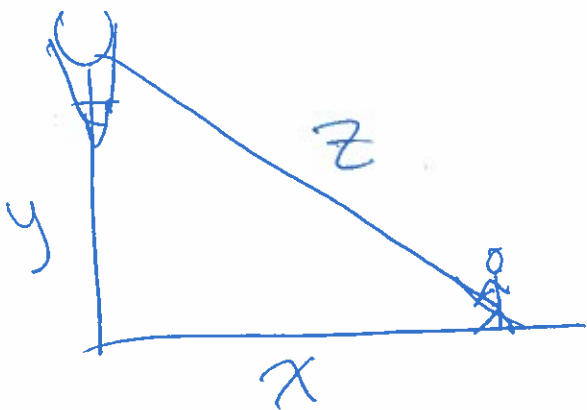
$$r = 100$$

$$\frac{dr}{dt} = 0.000238 \dots \text{cm/Sec}$$

Ex A balloon rises at 3 ft/sec 
 (rate = 3 ft/sec) You are 10 ft away
 (rate = 10 ft/sec) You walk away from
 the balloon at 4 ft/sec (rate = 4 ft/sec)

After two seconds (rate $t = 2$)

How fast is the distance between
 you and the balloon changing?



Pythagorean

$$z^2 = x^2 + y^2$$

Related Rate Problem

$$\frac{d}{dt}(z^2) = \frac{d}{dt}(x^2 + y^2)$$

$$2z \cdot \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$z \frac{dz}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$$

Rates
 $\frac{dz}{dt} = ?$

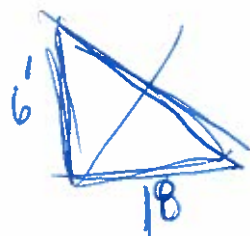
$$\frac{dx}{dt} = +4 \text{ ft/sec}$$

$$\frac{dy}{dt} = 3 \text{ ft/sec}$$

Values
 $x = 18$

$$y = 6$$

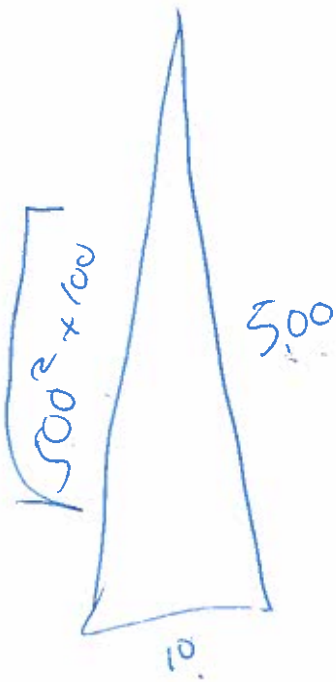
$$z = \sqrt{18^2 + 6^2} = 18.97 \dots$$



$$d = r t$$

$$3' / \text{sec} \cdot 2 \text{ sec} = 6'$$

$$z \frac{dz}{dt} = \frac{x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt}}{18.97 \approx z} = 4.74' / \text{sec}$$



Sell Balloons

Sales (Price)

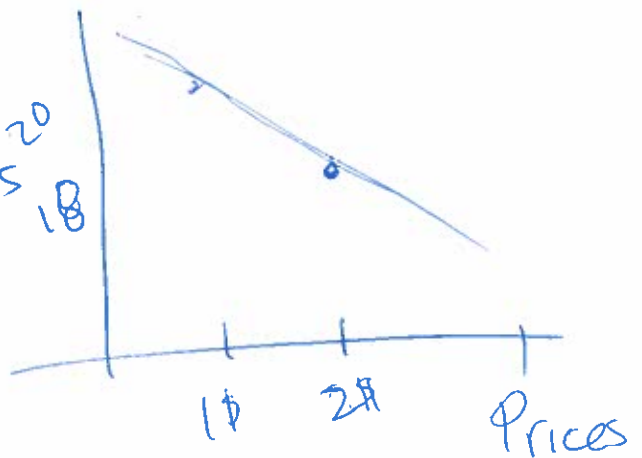
$$\text{Revenue} = \text{Price} \times \text{Sales}^2$$

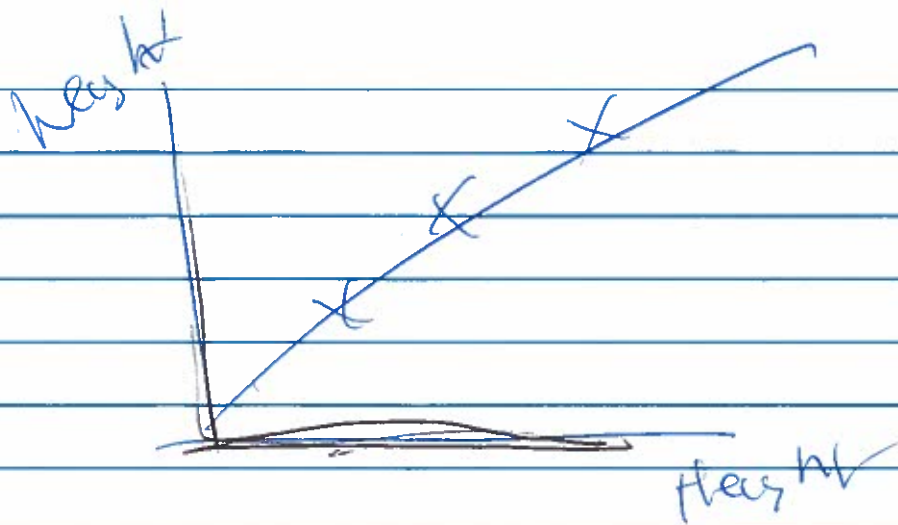
$$R = X \cdot (22 - 2X)$$

$$R = 22X - 2X^2$$

~~Revenue~~ ~~sales~~ balloons.

$$\frac{dR}{dt} = 22 \frac{dX}{dt} - 4X \cdot \frac{dX}{dt}$$

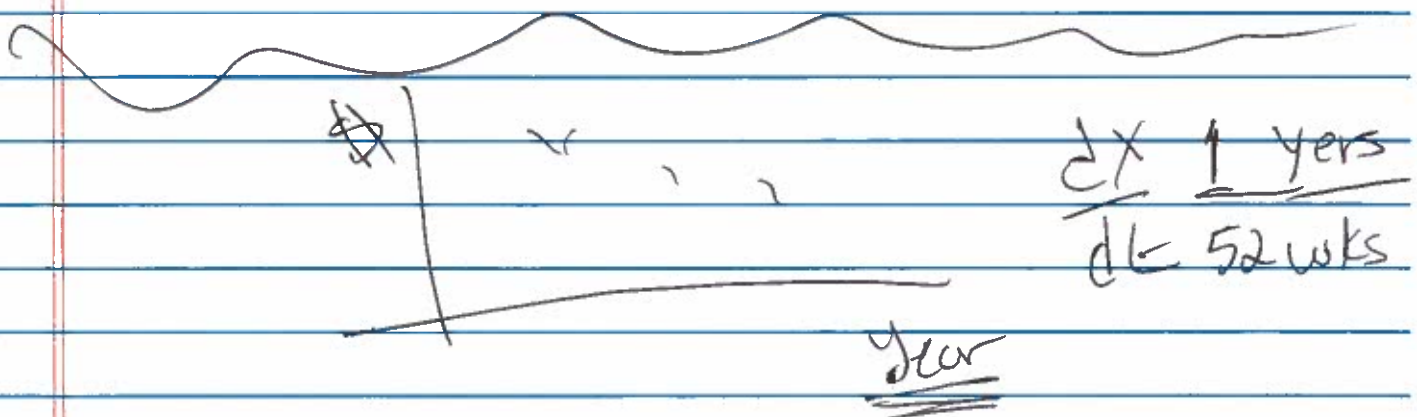




$$\text{Weight}(\text{Height}) = \text{reg. eq.}$$

$$\frac{dW}{dt} = n \cdot \text{deriv}(y, x) \cdot \frac{dx}{dt}$$

Height was growing 1" per year



$$y_1 = \text{reg. eq.}$$

$$y_2 = \text{deriv}(y, x) \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{24 \text{ hrs}}{1 \text{ days}}$$

GROUP NAME: (BEST FRIENDS) - (ELLIOT)

Date: 3-10-2014

Independent Variable (x-axis): YEARS

Dependant Variable (y-axis): ELECTRIC CAR SALES

Student Names (First and Last)

Speaker/Presenter: VINUTE AVHAD

Writer/Prep: LAUREN DORO

Leader/Collaborator: _____

Conclusion (in words):

IN 2014, WE ARE LOSING 4080 CAR SALES PER WEEK.

Supporting Work:

YEARS	SALES
2009	290292
2010	274555
2011	284064
2012	487480
2013	592192

x	y ₁	y ₂
2014,0	350222	-4080

Val. Rate

$$y = a \sin(bx + c) + d$$

$$a = 170314.8844$$

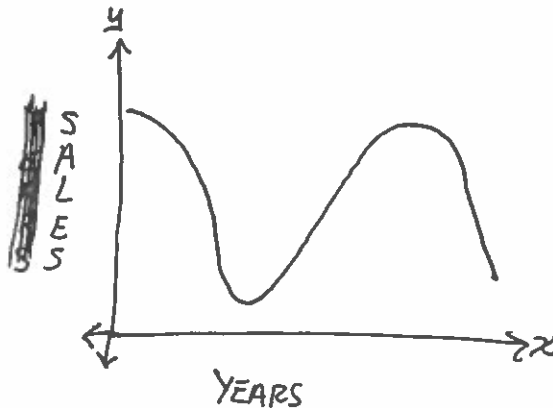
$$b = 1.272131339$$

$$c = -1.470607369$$

$$d = 384709.1247$$

$$y_1 = a \sin(bx + c) + d$$

$$y_2 = (\text{NDERIV}(y_1, x, x)) / 52$$



GROUP NAME:

Illuminati

Date: 03/10/14

Student Names (First and Last)

Speaker/Presenter: Ryan Piotrowski

Independent Variable (x-axis): years months

Writer/Prep: Bishop Bar

Dependant Variable (y-axis): gas prices

Leader/Collaborator: Danyan zhou

Conclusion (in words): gas prices are increasing at the rate of .15¢ per month

Supporting Work:

X	Y
1980	1
1990	1.54
2000	2.5
2008	3
2010	3.3

x12
months

$95 \times 12 =$

1114

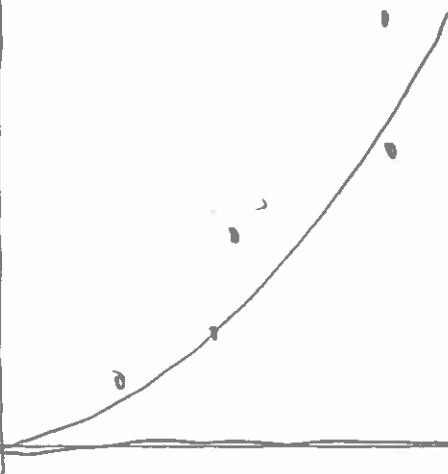
Exp Reg

$y = a \times b^x$

$y_1 = \exp(x)$

$y_2 = \text{nderv}(y_1, x) / 12$

X	y ₁	y ₂
2014	3.96	.15
2020	5.027	.199
2050	16.4	.65



GROUP NAME: We Mean Business I

Date: 3/10/14

Student Names (First and Last)

Speaker/Presenter: Christina Trujillo

Independent Variable (x-axis): Years

Writer/Prep: Simar Kalra

Dependant Variable (y-axis): Interest Rates

Leader/Collaborator: Yasmin Silverio

Conclusion (in words): In 2014, the interest rates would be growing by 1.75% per week

Supporting Work:

X	Y
2008	3.71
2009	1.53
2010	.29
2011	.38
2012	.58

$X = 2014$ ~~the year~~

$Y_1 R_{avg} = 4.924x^2 + -9.441x + 51.338$

$Y_2 = \text{deriv}(Y_1, X, 2014) / 52$

x	y ₁	y ₂
2014	4.4240	0.5267

GROUP NAME: Cha-Ching

Student Names (First and Last)

Date: 3/10/14

Speaker/Presenter: Trey Murrill

Independent Variable (x-axis): Years Days

Writer/Prep: Sheila Mae Gan

Dependant Variable (y-axis): Revenue

Leader/Collaborator: ~~Tatiana~~ Tatiana Calderon

Conclusion (in words):

~~In 2035 our club is making \$131,500 per ~~year~~ day~~
 In March 2016 our club ~~is~~ daily rate is 55,686 per day

Supporting Work:

L1	L2
13	35
12	27
11	26
10	17
9	16

~~$Y_1 = 4.8x + -28.6$~~
 ~~$Y_2 = nDeriv(Y_1, X, X) = 365$~~

X	Y1	Y2
35	139.4	0.1315
27	86.6	0.1315
3	-14.2	0.4315

$Y_1 = 18.65 \dots * \sin(-.647 \dots X + 2.320 \dots) + 24.27 \dots$

$Y_2 = nDeriv(Y_1, X, X) 90 / 365$

X	Y1	Y2
14	15.93	-2.665
16	5.9442	• 55688

GROUP NAME:

POLAR BEARZ

Date: _____

Student Names (First and Last)

Speaker/Presenter: Kausalya Munnim

Independent Variable (x-axis): year

Writer/Prep: Fremont Borela

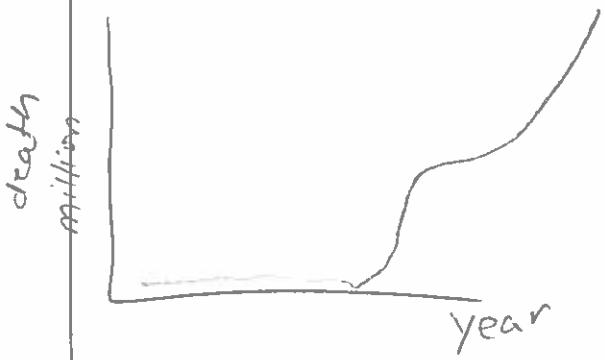
Dependant Variable (y-axis): deaths in millions due to HIV's

Leader/Collaborator: _____

Conclusion (in words):

per week .0151 millions of people die due to HIV in the year of 2014.

Supporting Work:



$y_1 = \text{Cubic Reg}$
 $y_2 = \text{ndenv}(y_1, x, x) \cdot \frac{1}{52}$

Table

x	y ₁	y ₂
2014	3.2199	.0151
	Value	Rate

<p>GROUP NAME: <u>Money Makers</u></p> <p>Date: <u>03/10/2014</u></p>	<p>Student Names (First and Last)</p> <p>Speaker/Presenter: <u>Bryan S.</u></p>
<p>Independant Variable (x-axis): <u>Years</u></p> <p>Dependant Variable (y-axis): <u>Crime rate in Detroit</u></p>	<p>Writer/Prep: <u>Edna Onyike</u></p> <p>Leader/Collaborator: <u>Monica K.</u></p>

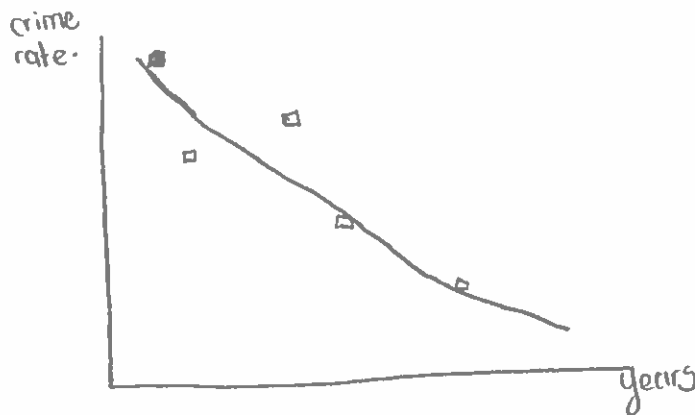
Conclusion (in words):
 In 2013, the crime rate is decreasing at $-.0011$ or -00.11% per week

Supporting Work:

$$Y' = \text{exponential reg} = 2.6539036900524 \times 10^{-8628704612988x}$$

$$Y' = (\text{deriv. } (y, x, x)) \frac{1}{52} = \text{" at year}$$

	x	y ₁	y ₂
2013		.3901	-.0011



GROUP NAME:	Student Names (First and Last)
Date: _____	Speaker/Presenter: <u>Shanon Isoe</u>
Independent Variable (x-axis): <u>years</u>	Writer/Prep: <u>Onur Turkan</u>
Dependant Variable (y-axis): <u>rates (sales)</u>	Leader/Collaborator: _____

Conclusion (in words): rates decreasing at .0706 monthly
In 2015 pink cars

Supporting Work:

x	Pink car Sales
01	50
05	46
08	34
11	55
12	51
14	29

Lin Reg.
 $Y_1 = -.8468x + 50.69$
 $Y_2 = \text{deriv}(Y_1, P, P) / 12$

x	Y ₁	Y ₂ (Rates)
JAN 15	37.996	-0.0706

Decades

rate of sales
 Sin Reg

x	Y ₁	Y ₂

x	Y ₁	Y ₂
JAN 15	46.92	0.01853 ← per month

Sin

x	Y ₁	Y ₂
15	46.92	0.2239

GROUP NAME: <u>Illuminati</u> Date: <u>3/5/14</u>	Student Names (First and Last) Speaker/Presenter: <u>Ryan Piotrowski</u> Writer/Prep: <u>Bishop Brer</u> Leader/Collaborator: <u>Danyan Zhou</u>
Independant Variable (x-axis): <u>years</u> Dependant Variable (y-axis): <u>gas prices</u>	

Conclusion (in words): When it reaches \$3.00 per gallon again, it will take 563 years to be to another \$1.

Supporting Work:

X	y
95	1
100	1.54
105	2.5
112	3
114	3.3

$114 - 95 = 19$ $2 \times 19 = 38 = \text{period}$
 Sincey Sincey $1, L_1, L_2, 3x$
 $y = a(\sin(bx + c) + d)$
 $a = 1.1$
 $b = .165$ $1.1 - 2.08$
 $c = 1.806$
 $d = 2.08$

$a = d$
 $1.1 - 2.08$ #'s between

X	y ₁	y ₂
3	328.03	563.6

years per dollar