Proving trigonometric identities using sum and difference properties

Prove the identity.

$$\boxed{\cos\left(x-\frac{\pi}{3}\right)}+\boxed{\sin\left(\frac{\pi}{6}-x\right)}=\cos x$$

$\boxed{\cos(x+y)}$ = cos x cos y - sin x sin y

cos(x-pi/3) = cos (-pi/3) cosx - sin (-pi/3) sin(x)

.5cosx + √3/2sinx

$\boxed{\sin(x+y)}$ = sinx cosy + cosx sin y

sin(pi/6-x)=sin(pi/6)cos(-x) + cos(pi/6) sin(-x)

.5 cos(x) − √3/2 · sin(x)

$\frac{1}{2}\cos x+\frac{1}{2}\cos x = 1\cos x$

Statement	Rule
$\sin\left(\frac{\pi}{3}+x\right)-\sin\left(\frac{\pi}{3}-x\right)$	
$=\left(\sin\frac{\pi}{3}\cos x+\cos\frac{\pi}{3}\sin x\right)-\left(\sin\frac{\pi}{3}\cos x-\cos\frac{\pi}{3}\sin x\right)$	Sum and Difference
$=\sin\frac{\pi}{3}\cos x+\cos\frac{\pi}{3}\sin x-\sin\frac{\pi}{3}\cos x+\cos\frac{\pi}{3}\sin x$	Algebra
$=2\cos\frac{\pi}{3}\sin x$	Algebra
$=2\cdot\frac{1}{2}\sin x$	Evaluation

Proving trigonometric identities using sum and difference properties

Prove the identity.

$$\sin(x+y)+\sin(x-y)=2\sin x\cos y$$

$\sin x\cos y+\sin y\cancel{\cos x}+\sin x\cos(-y)+\cos x\sin(-y)$

$\sin x\cos y-\cos x\cancel{\sin y}$

$\sin x\cos y+\sin x\cos y$

$2\sin x\cos y$

$$\frac{\boxed{\cot y}-\boxed{\cot x}}{\boxed{\cot x}\boxed{\cot y}-1}=\frac{\boxed{\frac{\cos y}{\sin y}}-\boxed{\frac{\cos x}{\sin x}}}{\boxed{\frac{\cos x}{\sin x}}\cdot\boxed{\frac{\cos y}{\sin y}}-1}\quad\begin{matrix}\cdot\ \sin x\sin y\\ \cdot\ \sin x\sin y\end{matrix}$$

Quotient identity for cotangent

$$=\frac{\dfrac{\sin x\cos y-\cos x\sin y}{\cancel{\sin x\sin y}}}{\dfrac{\cos x\cos y-\sin x\sin y}{\cancel{\sin x\sin y}}}$$

Algebra : Common denominator

$$=\frac{\sin x\cos y-\cos x\sin y}{\sin x\sin y}\cdot\frac{\sin x\sin y}{\cos x\cos y-\sin x\sin y}$$

Algebra : Multiply the numerator by the reciprocal of the denominator

$$=\frac{\sin x\cos y-\cos x\sin y}{\cos x\cos y-\sin x\sin y}=\frac{\sin(x-y)}{\cos(x+y)}$$

Algebra : Cancel common factor(s)

$$=\frac{\sin(x-y)}{\cos(x+y)}$$

Sum and difference identity

Proving trigonometric identities using sum and difference properties

Prove the identity.

$$\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$$

$(\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

FOIL

$(\sin x\cos y)^2 + \cancel{\cos x\sin y\sin x\cos y} - \cancel{\cos x\sin y\sin x\cos y} - (\cos x\sin y)^2$

$\sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$\cos^2 y = 1 - \sin^2 y \qquad \cos^2 x = 1 - \sin^2 x$

Py

$\sin^2 x(1-\sin^2 y) - ((1-\sin^2 x)\sin^2 y)$

$\sin^2 x - \cancel{\sin^2 x\sin^2 y} - \sin^2 y + \cancel{\sin^2 x\sin^2 y}$

$\sin^2 x - \sin^2 y$

Proving trigonometric identities using double-angle properties

Prove the identity.

$$\frac{\sin 2x}{1+\cos 2x} = \tan x$$

$\dfrac{2\sin x\cos x}{1+(1-\sin^2 x)}$

$\dfrac{2\sin x\cos x}{-\sin^2 x} = -2\cot x$

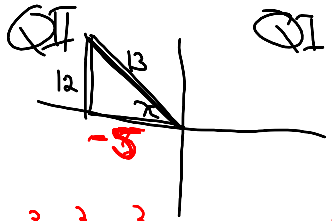
$\dfrac{2\sin x\cos x}{1+(\cos^2 x - \sin^2 x)}$

$\dfrac{2\sin x\cos x}{\cos^2 x + \cos^2 x}$

$\dfrac{2\sin x\cos x}{2\cos^2 x} = \tan x$

Double-angle identities: Problem type 1

Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ if $\sin x = \frac{12}{13}$ and x terminates in quadrant II.



SOH

$\sin = \frac{opp}{hyp} = \frac{12}{13}$

$?^2 + 12^2 = 13^2$

$?^2 = 25$

$? = \pm 5$

$\cos(2x) = \cos^2 x - \sin^2 x$

$= (\cos x)^2 - (\sin x)^2$

$= \left(\frac{-5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$

$\frac{25 - 144}{169} = \frac{-119}{169}$

$\sin(2x) =$

$2 \sin x \cos x$

$2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$

$= \frac{-120}{169}$

$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{-120}{-119} = \frac{120}{119}$

Double-angle identities: Problem type 2

Simplify the expression by using a double-angle formula.

$\cos^2 5\theta - \sin^2 5\theta = \cos(2 \cdot 5\theta) = \cos(10\theta)$

The following identities are the double-angle formulas.

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

More on these formulas

Double-angle identities: Problem type 2

Simplify the expression by using a double-angle formula.

$$\cos^2\frac{2\pi}{9}-\sin^2\frac{2\pi}{9}$$ =cos(2*2pi/9)=cos(4pi/9)

The following identities are the double-angle formulas.

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ \cos 2x &= \cos^2 x-\sin^2 x\\ &= 1-2\sin^2 x\\ &= 2\cos^2 x-1\\ \tan 2x &= \frac{2\tan x}{1-\tan^2 x}\end{aligned}$$

More on these formulas

Double-angle identities: Problem type 2

Simplify the expression by using a double-angle formula.

$$\frac{2\tan\frac{\pi}{11}}{1-\tan^2\frac{\pi}{11}}$$

$$\boxed{\tan 2\frac{\pi}{11}}=\frac{2\tan\frac{\pi}{11}}{1-\tan^2\frac{\pi}{11}}$$

replace x=pi/11

tan(2 pi/ 11)

The following identities are the double-angle formulas.

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ \cos 2x &= \cos^2 x-\sin^2 x\\ &= 1-2\sin^2 x\\ &= 2\cos^2 x-1\\ \tan 2x &= \frac{2\tan x}{1-\tan^2 x}\end{aligned}$$

More on these formulas

Statement	Rule
$\frac{\sin 2x}{1+\cos 2x}$	
$= \frac{2\sin x\cos x}{1+\left(2\cos^2 x-1\right)}$	Double-angle
$= \frac{2\sin x\cos x}{2\cos^2 x}$	Algebra
$= \frac{\sin x}{\cos x}$	Algebra
$= \tan x$	Quotient

ODD & EVEN

ODD

$f(x) = x^3$

$f(-x) = (-x)^3 = -x^3$

$f(-x) = -f(x)$

$\sin(-x) = -\sin x$

$\sin(-\pi/3) = -\sin(\pi/3)$

EVEN

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

$f(-x) = f(x)$

$\cos(-x) = \cos(x)$

$\cos(-\pi/3) = \cos(\pi/3)$