Finding values of trigonometric functions given information about an angle: Problem type 1

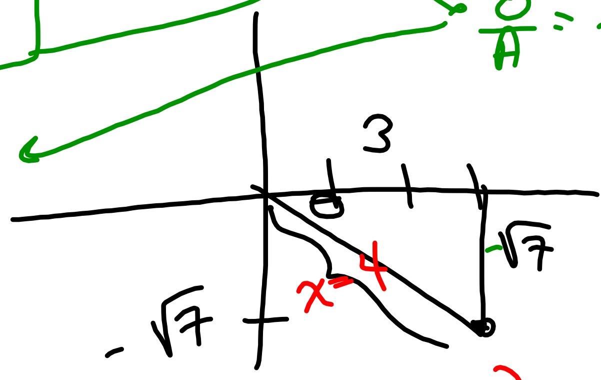
Let $(3, -\sqrt{7})$ be a point on the terminal side of θ.

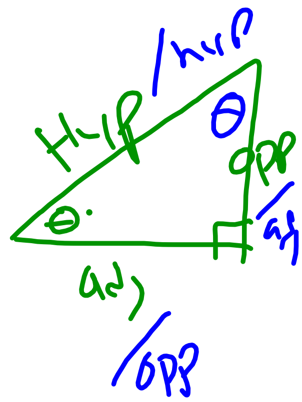
Find the exact values of $\cos\theta$, $\csc\theta$, and $\tan\theta$.

SOH
CAH
TOA

$\frac{A}{H} = \frac{3}{4}$

$\frac{O}{A} = \frac{-\sqrt{7}}{3}$





Pythagorean Theorem

$3^2 + (\sqrt{7})^2 = x^2$

$16 = x^2$

$4 = x$

$\csc\theta = \frac{1}{\sin\theta} = \frac{H}{O} = \frac{-4}{\sqrt{7}}$

$\sin\theta = \frac{O}{H}$

Finding values of trigonometric functions given information about an angle: Problem type 2

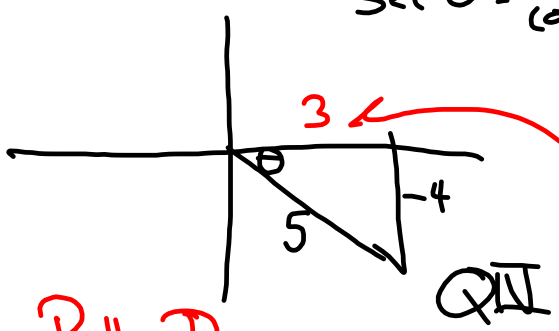
Let θ be an angle in quadrant IV such that $\sin\theta = -\frac{4}{5}$. $= \frac{O}{H} = \frac{-4}{5}$

Find the exact values of $\sec\theta$ and $\cot\theta$.

$\sec\theta = \frac{1}{\cos\theta} = \frac{H}{A} = \frac{5}{3}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{A}{O}$

$= \frac{3}{-4}$



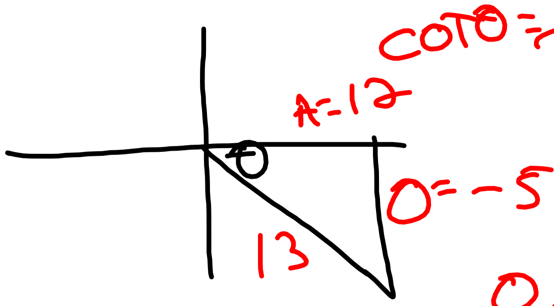
Pyth. Theorem

Finding values of trigonometric functions given information about an angle: Problem type 3

Let θ be an angle in quadrant IV such that $\cot\theta = -\frac{12}{5}$.

Find the exact values of $\sin\theta$ and $\sec\theta$.

$\cot\theta = \frac{1}{\tan\theta} = \frac{A}{O}$



Pyth

$\sin\theta = \frac{O}{H} = -\frac{5}{13}$

$\sec\theta = \frac{H}{A} = \frac{13}{12}$

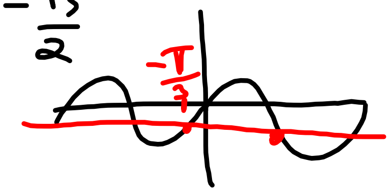
Values of inverse trigonometric functions

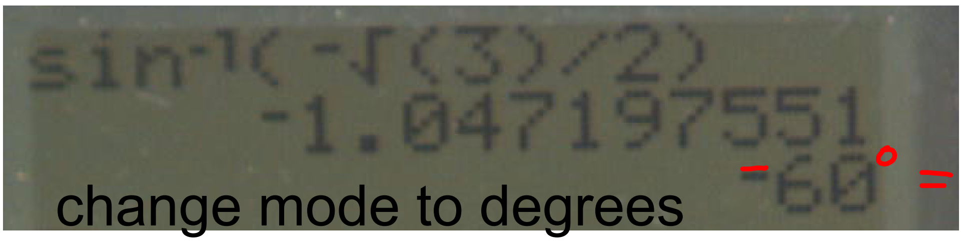
Find the exact value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

Write your answer in radians in terms of π.

$\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\sin\theta = -\frac{\sqrt{3}}{2}$





$= -\pi/3$

$\cancel{60} \cdot \frac{\pi}{\cancel{180}_3}$

$$\frac{\sin x}{\cos x} = \tan x$$

Day 19 - Question #3;
Simplifying trigonometric expressions

Simplify.

$$\frac{\sec x}{\tan x \csc x} = \frac{\frac{1}{\cos x}}{\frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}}} = \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}} = \underline{1}$$

Algebra

Use algebra and the fundamental trigonometric identities.
Your answer should be a number or use a single trigonometric function.

Reduce to $\sin x$, $\cos x$

$\sec x = \frac{1}{\cos x}$

$\csc x = \frac{1}{\sin x}$

$\tan x = \frac{\sin x}{\cos x}$

Equations===3 possibilities

solve for answer - condition

x+1=x+2 contradiction

x+1=x+1 identity TRUE ALWAYS

Reciprocal Identities:

(1) $\sin u = \dfrac{1}{\csc u}$ (2) $\cos u = \dfrac{1}{\sec u}$

(3) $\tan u = \dfrac{1}{\cot u}$ (4) $\csc u = \dfrac{1}{\sin u}$

(5) $\sec u = \dfrac{1}{\cos u}$ (6) $\cot u = \dfrac{1}{\tan u}$

Quotient Identities:

(1) $\tan u = \dfrac{\sin u}{\cos u}$

(2) $\cot u = \dfrac{\cos u}{\sin u}$

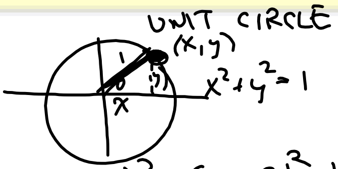
Pythagorean Identities:

(1) $\sin^2 u + \cos^2 u = 1$

(2) $1 + \tan^2 u = \sec^2 u$

(3) $1 + \cot^2 u = \csc^2 u$

$x = \cos\theta$
$y = \sin\theta$



$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

Divide by $\cos^2\theta$

$1 + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

Statement	Rule
$\sec^2 x - \tan^2 x$	
$= \left(1 + \tan^2 x\right) - \tan^2 x$	Pythagorean
$= 1$	Algebra

$\left(1 + \cot^2 x\right)\tan^2 x$	
$= \csc^2 x \tan^2 x$	Pythagorean
$= \left(\dfrac{1}{\sin^2 x}\right)\tan^2 x$	Reciprocal
$= \left(\dfrac{1}{\sin^2 x}\right)\left(\dfrac{\sin^2 x}{\cos^2 x}\right)$	Quotient
$= \dfrac{1}{\cos^2 x}$	Algebra
$= \sec^2 x$	Reciprocal

$$\tan x\left(1+\cot^2 x\right)=\frac{1}{\cos x\,\sin x}$$

Statement	Rule
$\tan x\left(1+\cot^2 x\right)$	
$= \tan x\left(\csc^2 x\right)$	Pythagorean
$= \tan x \cdot \dfrac{1}{\sin^2 x}$	Reciprocal
$= \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin^2 x}$	Quotient
$= \dfrac{1}{\cos x\,\sin x}$	Algebra
Thank you, your proof is complete; Click on **Done** below to submit your answer.	

Day 19 - Question #8;
Proving trigonometric identities: Problem type 3

Prove the identity.

$$\frac{\tan x-\cot x}{\tan x+\cot x}=\sin^2 x-\cos^2 x$$

Statement	Rule
$\dfrac{\tan x-\cot x}{\tan x+\cot x}$	
$= \dfrac{\dfrac{\sin x}{\cos x}-\dfrac{\cos x}{\sin x}}{\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}}$	Quotient
$= \dfrac{\dfrac{\sin x}{\cos x}-\dfrac{\cos x}{\sin x}}{\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}} \dfrac{\cos x\,\sin x}{\cos x\,\sin x}$	Algebra
$= \dfrac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x}$	Algebra
$= \sin^2 x-\cos^2 x$	Pythagorean

Day 19 - Question #7;
Proving trigonometric identities: Problem type 2

Prove the identity.

$$\frac{\cos^2 x}{(1+\sin x)^2} = \frac{1-\sin x}{1+\sin x}$$

Statement	Rule
$\frac{\cos^2 x}{(1+\sin x)^2}$	
$= \frac{\cos^2 x}{(1+\sin x)^2} \cdot \frac{1-\sin x}{1-\sin x}$	Algebra
$= \frac{\cos^2 x}{(1+\sin x)(1+\sin x)} \cdot \frac{1-\sin x}{1-\sin x}$	Algebra
$= \frac{\cos^2 x}{(1+\sin x)} \cdot \frac{1-\sin x}{1-\sin^2 x}$	Algebra
$= \frac{\cos^2 x}{(1+\sin x)} \cdot \frac{1-\sin x}{\sin^2 x+\cos^2 x-\sin^2 x}$	Pythagorean
$= \frac{\cos^2 x}{(1+\sin x)} \cdot \frac{1-\sin x}{\cos^2 x}$	Algebra
$= \frac{1-\sin x}{1+\sin x}$	Algebra