

1. Definition  $y = b^x$  if and only if  $x = \log_b y$

Forward: **Solve for an exponent**

EX: Find the inverse for  $y = 4^x$

Two steps to inverse: 1. Solve for x 2. Switch x/y

1.  $X = \log_{\text{base}=4}(\text{argument}=y)$  by prop 1
2.  $Y^{-1} = \log_4(X)$

EX: The How long will it take for an investment to double at 5% interest per year?  $P = Qe^{(RT)}$

$P = Qe^{RT}$   
 $2Q = Qe^{(.05T)}$  so  $2 = e^{.05T}$  so by prop 1  $.05T = \ln(2)$   
 By algebra  $T = \ln(2)/.05 = 13.86$  years

Math 0: Solver

$$0 = P - Qe^{(RT)}$$

$$P = 200$$

$$Q = 100$$

$$R = .05$$

$$T = 0 < \text{alpha} > (\text{enter})$$

$$\text{④ } T = 13.86.$$

$$.05T = \log_e(2)$$

$$\frac{.05T}{.05} = \frac{\ln(2)}{.05}$$

$$T = \frac{\ln 2}{.05}$$

Reverse: "Get rid of a log"

EX: Find  $x$  if  $5 = \log(2x)$  (base=10)

Want to GET RID OF LOG

Start with base 10 exponent =  $10^5 = 2x$  by prop 1

$10000 = 2x$  by algebra  $x = 5000$

Ex  $3 - \log_2(x+1) = 7$

$$\begin{array}{r} 3 - \log_2(x+1) = 7 \\ -3 \qquad \qquad -3 \\ \hline -\log_2(x+1) = 4 \\ \frac{-1}{-1} \qquad \frac{4}{-1} \\ \hline \log_2(x+1) = -4 \end{array}$$

Prop 1  $\rightarrow 2^{-4} = x+1 \quad x = 2^{-4} - 1$

## 2. Sum/Product Property

Forward: **Combine logs**  $\log A + \log B = \log(AB)$

EX: Find a single log for

$$\ln 2 + \ln 5 = \ln(10) \text{ by prop 2}$$

$$\log 25 + \log 4 = \log(100) \text{ by prop 2 or } \underline{\underline{2}}$$

Reverse: **Find the components of a log**

EX: Use table of logs to find

$$1.1 \times 10^1$$

$$\log 11 = \log(10 \cdot 1.1) = \log(10) + \log(1.1)$$

$$= 1 + 0.0414 = 1.0414$$

$$\log(12000) = \log(1.2 \cdot 10^5) = \log(1.2) + \log(10^5)$$

$$= 5 + 0.0792 = 5.0792$$

TABLE

log	.1	.2	.3	.4
0	-1	.69		

3. "Ladder" Property  $n \log_b x = \log_b x^n$ Forward: **Get rid of a coefficient**EX: Find  $3 \log 4 + 2 \log 5$ 

$$= \log 4^3 + \log 5^2 \text{ by prop 3.}$$

$$= \log(4^3 \cdot 5^2) \text{ by prop 2}$$

Reverse: **Get rid of an exponent inside a log**

EX: Find  $\log_2 4^{50000}$   
 $= 50000 \log_2 4$  by prop 3  $= 50000 * 2 = 100000$

$$\log_2 4 = x^2$$

$$2^x = 4$$

Change of Base  $\log_b x = \frac{\log_a x}{\log_a b}$

Forward: **Evaluate a old bases in terms of a new one**

EX: Find  $\log_2 4 =$  we have ln and log in calculator

$\log_2 4 = \frac{\log(4)}{\log(2)} = 2 = \frac{\ln(4)}{\ln(2)}$  By prop 4

$$\log_2 4 = \frac{\log(4)}{\log(2)}$$

$$= \frac{\ln(4)}{\ln(2)}$$

Reverse: Divide logs of a same base to get a single log

EX: Simplify  $\ln(100)/\ln 10 = \log(100)$  by prop 4

5. Log of Both Sides  $x = y$  iff  $\log_b x = \log_b y$

Forward: Take the log of both sides

Reverse: Drop the logs from both sides

EX: If  $\log x = \log 40$  find  $x$ .

Then  $x = 40$  by prop 5

[CLOSE WINDOW](#)

**Converting between logarithmic and exponential equations**

Rewrite as a logarithmic equation.

$$3^0 = 1$$

$$\boxed{0} = \log_{\boxed{3}} \boxed{1}$$

For any numbers  $a$ ,  $b$ , and  $c$ , with  $a$  and  $c$  positive ( $a \neq 1$ ), we have

Your resources will appear

**Converting between natural logarithmic and exponential equations**

Rewrite as a logarithmic equation.

$$e^y = 6$$

$$\boxed{y} = \log_{\boxed{e}} \boxed{6}$$

$$y = \ln 6$$

**Evaluating a logarithmic expression**

Evaluate.

$$\log_2 \frac{1}{4}$$

$$\rightarrow \log_2(1/4) / \log_2(2)$$

Explain

$$\log_2(2^{-2}) = x$$

$$2^{-2} = 2^x \quad x = -2$$

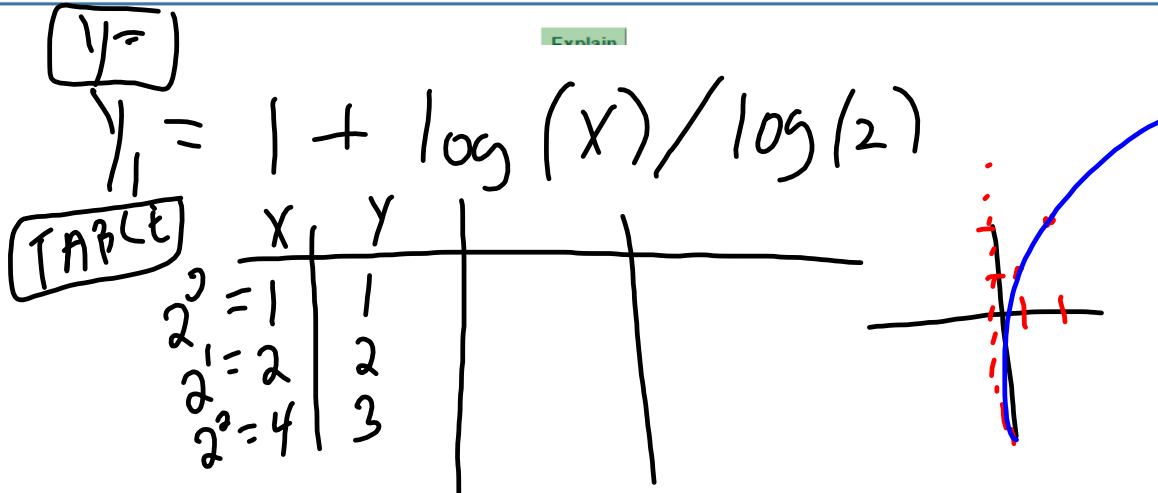
## Graphing a logarithmic function: Basic

Graph  $g(x) = 1 + \log_2 x$ .

To graph the function, plot at least two points on the graph, draw all asymptotes, and then click on the graph icon.

$$\log_2 4 = 2$$

$$\log_2 8 = 3$$

[CLOSE WINDOW](#)

## The graph, domain, and range of a logarithmic function

Graph the function  $g(x) = \log_4(x - 3)$  and give its domain and range using interval notation.

Can't take log  
of anything that's NOT

$x - 3 > 0$

Domain  $(3, \infty)$

Range  $(-\infty, \infty)$

### Basic properties of logarithms

Fill in the missing values to make the equations true.

$$(a) \log_3 4 - \log_3 11 = \log_3 \boxed{4/11}$$

$$\log_3 4 + \log_3 11^{-1}$$

$$\log_3 4 \cdot 11^{-1}$$

$$(b) \log_7 9 + \log_7 \boxed{5} = \log_7 45$$

$$\log_3 (4/11)$$

$$(c) \log_6 32 = \boxed{5} \log_6 2$$

$$\log_6 2^5 = \boxed{5} \log_6 2$$

↑.↙

### Writing an expression as a single logarithm

Write the expression as a single logarithm.

$$7\log_4 w - \frac{1}{5}\log_4 y + 2\log_4 z$$

P3

$$P3 \quad \log_4 w^7 + \log_4 y^{-1/5} + \log_4 z^2$$

$$P2 \quad \log_4 (w^7 \cdot y^{-1/5} z^2)$$



$$\underline{\text{Ex}} \quad 5 = 6^x$$

Solve for  $x$

$$P1 \quad x = \log_6 5 = \frac{\log(5)}{\log(6)}$$

**Finding the time to reach a limit in a word problem on exponential growth or decay**

A laptop computer is purchased for \$2700. Each year, its value is 80% of its value the year before. After how many years will the laptop computer be worth \$900 or less? (Use the calculator provided if necessary.)

Write the *smallest possible* whole number answer.

$$2700(.80) = 2160$$

$$2700(.80)^x = 900$$

$$y_1 = 900$$

$$y_2 = 2700(.80)^x$$

$$(.80)^x = \frac{900}{2700}$$

$$x = \log_{.80} \left( \frac{1}{3} \right)$$

$$x = \frac{\log(1/3)}{\log(.8)} \\ = 4.9$$