Sum and Difference Formulas

$$\boxed{\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)}$$

$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

EX $\sin(30° + 45°) = \sin(30°)\cos(45°) + \cos(30°)\sin(45°)$

$\sin(75°) = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

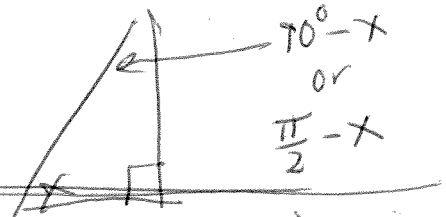
$= .9659 \Rightarrow \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\sin 75° = .9659\ldots$

EX $\sin\left(\frac{\pi}{2} - x\right) = \sin\left(\frac{\pi}{2}\right)\cos(-x) + \cos\left(\frac{\pi}{2}\right)\sin(-x)$

$= 1 \cdot \cos(x) + 0 \cdot \sin(x)$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$



EX $\sin(a+a) = \sin(a)\cos(a) + \cos(a)\sin(a)$

$$\boxed{\sin(2a) = 2\sin(a)\cos(a)}$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

Ex $\cos(15^\circ) = \cos(45)\cos(30) + \sin(45^\circ)\sin(30^\circ)$

$\cos(45^\circ - 30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = .9659\ldots$ Calculate $\cos(15^\circ)$

Ex $\cos(x+x) = \cos x \cos x - \sin x \sin x$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos(2x) = 1 - \sin^2 x - \sin^2 x$

$$\cos(2x) = 1 - 2\sin^2 x$$

$\cos(2x) = \cos^2 x - (1 - \cos^2 x)$

$$\cos(2x) = 2\cos^2 x - 1$$

$$\frac{\cos(2x) + 1}{2} = \cos^2 x$$

$$\frac{-\cos(2x) + 1}{2} = \sin^2 x$$

$$\tan(a+b) = \frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin(a)\cos(b)+\cos(a)\sin(b)}{\cos(a)\cos(b)-\sin(a)\sin(b)}$$

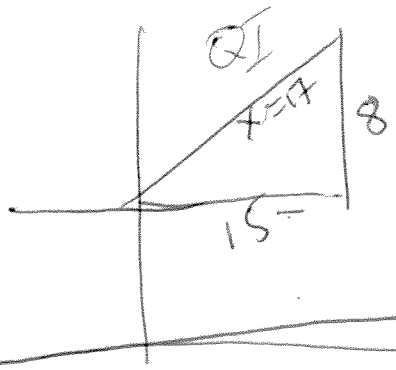
$1 + \tan(\quad)$

$$\sec(a+b) = \frac{1}{\cos(a+b)}$$

$\tan\theta = \frac{8}{15}$ opp, QI, adj

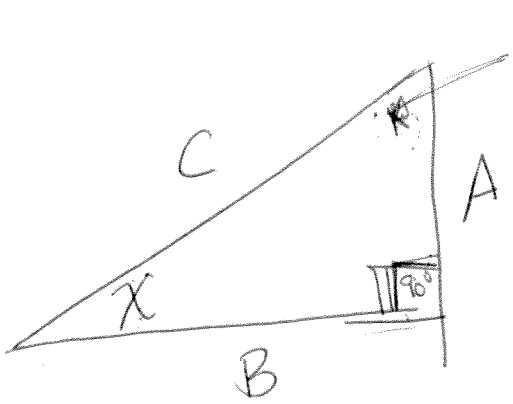
$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$



$8^2 + 15^2 = x^2$

$x = 17$



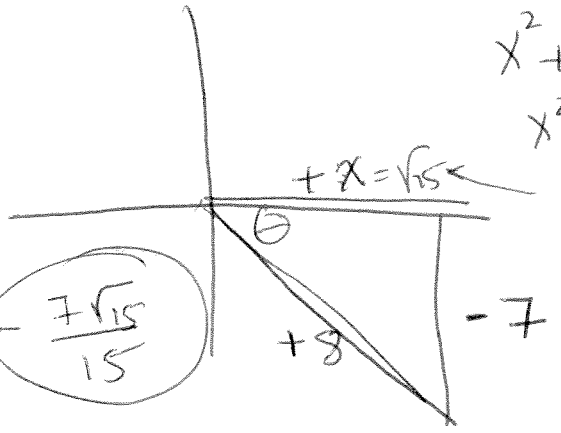
$$\sin(x) = \frac{opp}{hyp} = \frac{A}{C}$$

or

$$\cos\left(\frac{\pi}{2} - x\right) = \frac{adj}{hyp} = \frac{A}{C}$$

$$\csc\theta = -\frac{8}{7} \qquad \theta \text{ is QIV}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{opp}{hyp}} = \frac{hyp}{opp}$$

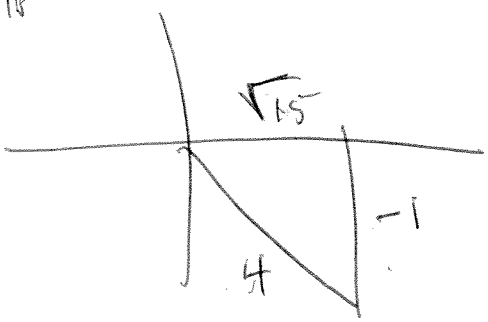


$$x^2 + 49 = 64$$
$$x^2 = 15$$
$$x = \sqrt{15}$$

$$\tan\theta = \frac{opp}{adj} = \frac{-7}{\sqrt{15}} = -\frac{7\sqrt{15}}{15}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{15}}{8}$$

$$\sin\theta = -\frac{1}{4} = \frac{opp}{hyp} \qquad \theta \text{ in QIV}$$



$$1^2 + x^2 = 4^2$$
$$x^2 = 16 - 1$$

Identity True Always (for all values of x)

Reciprocal Identities

* $\sec(x) = \frac{1}{\cos(x)}$
* $\csc(x) = \frac{1}{\sin(x)}$
* $\cot(x) = \frac{1}{\tan(x)}$

Quotient Identity

* $\tan(x) = \frac{\sin(x)}{\cos(x)}$
* $\cot(x) = \frac{\cos(x)}{\sin(x)}$

Pythagorean Identity

* $\sin^2(x) + \cos^2(x) = 1$

Divide by $\sin^2(x)$:

$1 + \cot^2(x) = \csc^2(x)$

Divide by $\cos^2(x)$:

$\tan^2(x) + 1 = \sec^2(x)$

$\sin^2(\text{Frane}) + \cos^2(\text{Frane}) = 1$

$\sin^2(0) + \cos^2(0) = 1$

$\sin^2(x) = (\sin x)^2 = \sin x \cdot \sin x$

NOT

$\sin(x^2) = \sin x^2$

$\sin^{-1}(x)$ = inverse function

$\sin^{-2}(x) = \frac{1}{\sin^2(x)} = \left(\frac{1}{\sin x}\right)^2 = \csc^2(x)$

$\sin^2(x) = 1 - \cos^2(x)$ *

$\cos^2(x) = 1 - \sin^2(x)$ *

$\cot^2(x) = \csc^2(x) - 1$

$\tan^2(x) = \sec^2(x) - 1$

Ex $f(x) = \tan^2(x) \cdot \cos^2(x)$

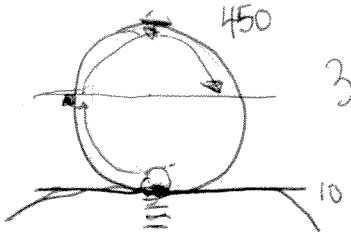
Quotient.

$= \frac{\sin^2(x)}{\cos^2(x)} \cdot \frac{\cos^2(x)}{1}$

Algebra

$= \sin^2(x)$

London Eye



30 mins

Time	Height
0	10
7.5	230
15	450
22.5	230
30	10

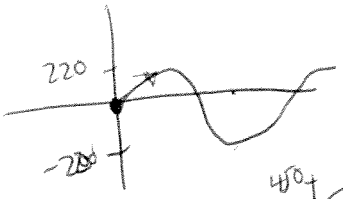
460 ÷ 2 = 230

STAT EDIT

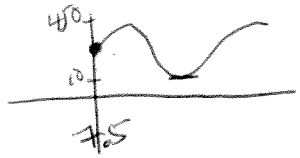
L1	L2
0	10
7.5	230
15	450
22.5	230
30	10

STAT > CALC C: SinReg

$y = \underset{A}{220}\sin(\underset{B}{.20}x - \underset{C}{1.57}) + \underset{D}{230}$



up 230



Period = 30 min $= \frac{2\pi}{B}$

$B = \frac{2\pi}{30}$

Amp $= 450 - 230 = 220 = A$ $\quad \frac{\text{Top} - \text{Bottom}}{2}$

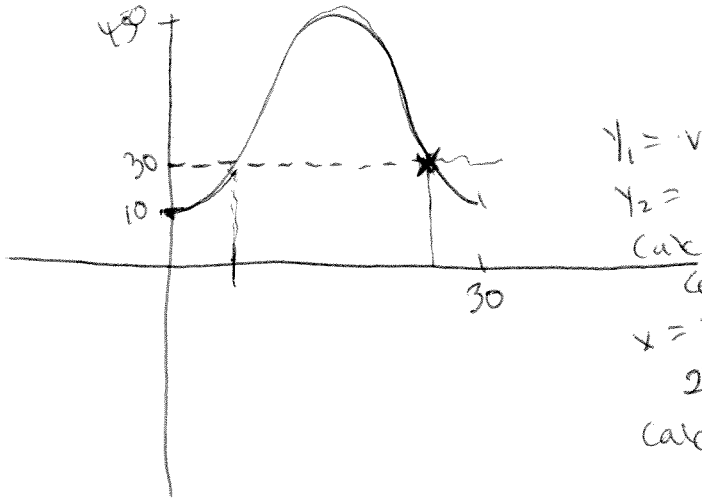
Raised up by $D = \frac{\text{Top} + \text{Bottom}}{2}$

$\frac{450+10}{2} = 230$

Phase Shift $7.5 = -\frac{C}{B}$

$C = -7.5(B) =$

$= -7.5\left(\frac{2\pi}{30}\right) = 1.57\ldots$



$Y_1 =$ VARS 5: (enter)

$Y_2 = 30$

Calc 5 Intersect

(enter)(enter)(guess)

$x = 27.94$

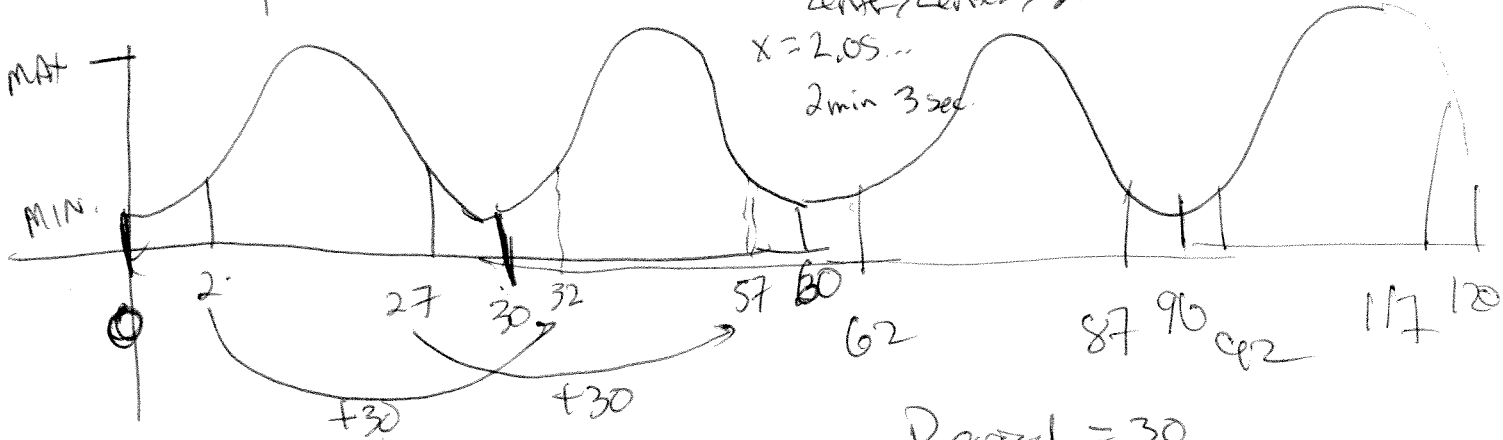
27 min 56 sec

Calc 5 Intersect

(enter)(enter)(guess)

$x = 2.05\ldots$

2 min 3 sec



Period = 30

All solution = Unique Solution $2 + 30n$ $\quad n$ is integer

$27 + 30n$

EX $19\sin(x+1)-8=10$

$+8 \quad +8$

$\frac{19\sin(x+1)}{19} = \frac{18}{19}$

$\sin(x+1) = \frac{18}{19}$ use inverse.

$x+1 = \sin^{-1}\left(\frac{18}{19}\right)$

$y = \sin^{-1}(x)$

$x = -1 + \sin^{-1}\left(\frac{18}{19}\right)$ $\sin y = x$

$x = .244\ldots$

Does $\sin(x)$ have inverse?

NO

$$\sin^{-1}\left(-\frac{1}{2}\right) = x$$

$$-\frac{1}{2} = \sin x$$

$-\pi/6$		$\pi/6$
$-1/2$	0	$1/2$