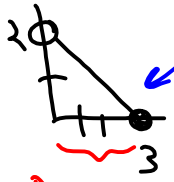
Precalc = Study of Functions

Data Graphs ← Equations
→ Plots

Job ⇒ In = Domain
Out = Range

x	y
1	3
2	4

D: 1, 2
R: 3, 4



D: (1, 3]
R: [0, 2)

Equations

Piecewise

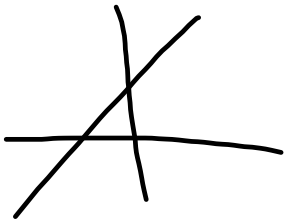
$f(x) = \{$

$\{$ Linear
Quad
Cubic $\Rightarrow$ Polynomial
Degree
Lead

Lines: $y = mx + b$

Degree: 1

Lead: m

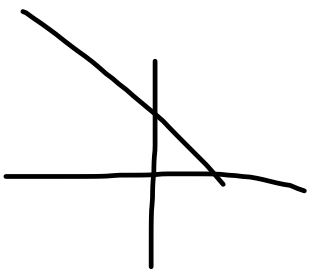


END: Disco

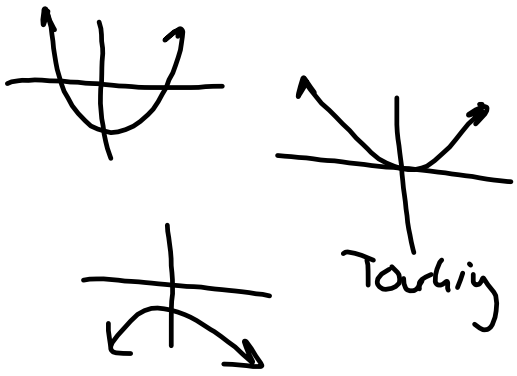
Zeros: 1

Face: 1

Turns: 0

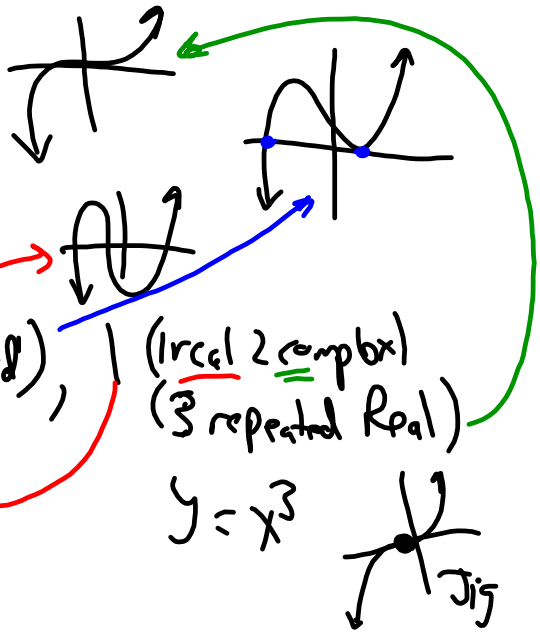


Parabolas $y = Ax^2 + Bx + C$



Degree: 2
Lead: A
END: Happy/Sad
Zeros: 2, 1 (repeated), 0 (2 complex)
Faces: 2
Turning: 1

Cubic $y = Ax^3 + Bx^2 + Cx + D$



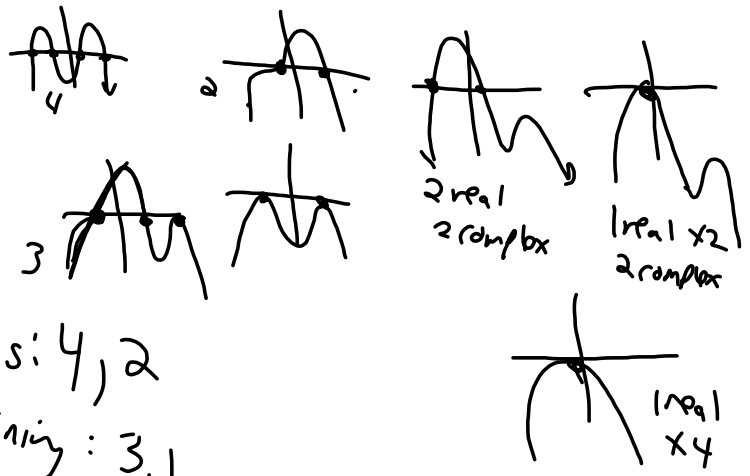
Degree (all): 3
Lead: A
END: Disco
Zeros: 3, 2 (1 repeated), 1 (1 real 2 complex) (3 repeated Real)
$y = x^3$

Quintic

Degree: 4 (even)

END: Happy/Sad

Zeros: 4 (distinct), 3 (2 distinct 1 repeat), 2

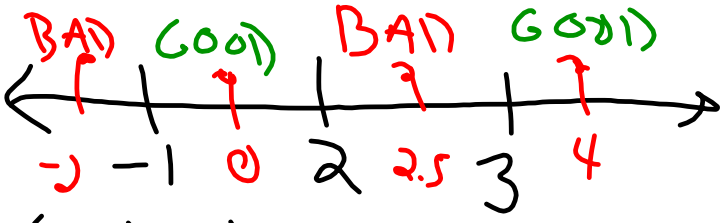


Faces: 4, 2

Turning: 3, 1

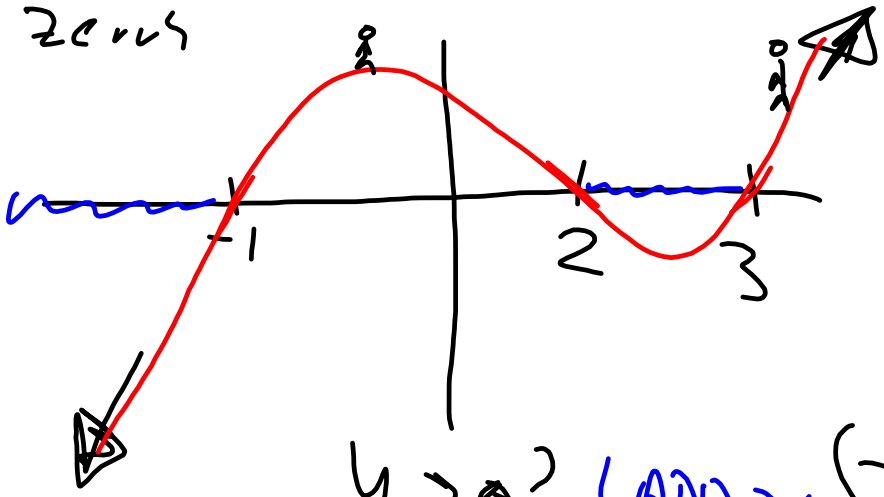
Polynomial Inequalities

$= (x+1)(x-2)(x-3) > 0$



x	y = (x+1)(x-2)(x-3)
-2	-24 = (-1)(-4)(-6) > 0? No
0	6 = (1)(-2)(-3) > 0! Yes
2.5	=
4	+

$y = (x+1)^1(x-2)^1(x-3)^1$ ← PASS



$y > 0$? LAND > 0 $(-1,2) \cup (3,\infty)$

$y \geq 0$? LAND + Both $[-1,2] \cup [3,\infty)$

Rational Functions $y = \frac{P}{Q}$

Zeros: ZN PASS OR TOUCH

Vert. Asymptes: ZD HEART OR VOLCANO

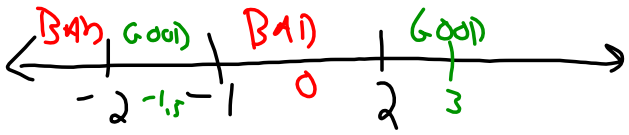
END DN < DD
HA: $y = 0$
$\frac{4}{x}$

END DN = DD
HA: $y = LN/LD$
$y = \frac{4x}{x+1}$
$y = 4$

END DN > DD
SLANT
END OF SLANT
DR = DN - DD
ODD: Disco
EVEN: Happy/SAD
LR: LN/LD
POS or NEG
$y = x^3$

Rational Inequalities

$$y = \frac{x+1}{x^2-4} = \frac{x+1}{(x-2)(x+2)} \geq 0$$



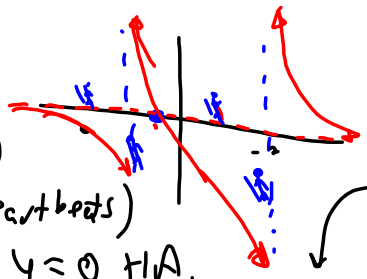
$y = \frac{x+1}{(x-2)(x+2)}$

Zero: -1 (pass)

VA: 2, -2 (Heartbeats)

DNE END $y = 0$ HA.

$y(-3) = \frac{(-2)}{(-5)(-1)} = \ominus$



ABOVE HORIZONTAL BEACH

$(-2, -1] \cup (2, \infty)$

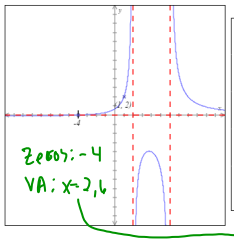
DON'T DO WITH ALGEBRA

$$\frac{x+1}{x^2-4} \geq 0$$

Writing the equation of a rational function given its graph

The figure below shows the graph of a rational function f with vertical asymptotes $x = 2$, $x = 6$, and horizontal asymptote $y = 0$. The graph also has an x-intercept of -4, and it passes through the point $(1,2)$.

The equation for $f(x)$ has one of the five forms shown below. Choose the appropriate form for $f(x)$, and then write the equation. You can assume that $f(x)$ is in simplest form.



- $f(x) = \frac{a}{x-b}$
- $f(x) = \frac{a(x-b)}{x-c}$
- $f(x) = \frac{a}{(x-b)(x-c)}$ ✓
- $f(x) = \frac{a(x-b)}{(x-c)(x-d)}$ ✓ — 1 zero, 2 VA, HA: $y=0$
- $f(x) = \frac{a(x-b)(x-c)}{(x-d)(x-e)}$

2 D: 2, 6

Denominator Factors $(x-2)(x-6)$

HA: $y=0$

DD > DN

2 N: -4

Numerator Factors

~~Factors $(x+4)(x+4)$~~

Factor $(x+4)$

$y = \frac{a(x+4)}{(x-2)(x-6)}$

Point $(1,2)$

$2 = \frac{a(5)}{(-1)(-5)}$

$2 = a$

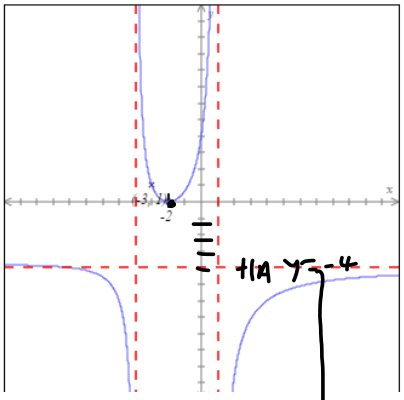
fn's:

$y = \frac{2(x+4)}{(x-2)(x-6)}$

Writing the equation of a rational function given its graph

The figure below shows the graph of a rational function f with vertical asymptotes $x = 1$, $x = -4$, and horizontal asymptote $y = -4$. The graph also has an x-intercept of -2, and it passes through the point $(-3,1)$.

The equation for $f(x)$ has one of the five forms shown below. Choose the appropriate form for $f(x)$, and then write the equation. You can assume that $f(x)$ is in simplest form.



- ~~$f(x) = \frac{a}{x-b}$~~ 2 VA
- ~~$f(x) = \frac{a(x-b)}{x-c}$~~ 2 VA
- ~~$f(x) = \frac{a}{(x-b)(x-c)}$~~ 1 zero
- $f(x) = \frac{a(x-b)}{(x-c)(x-d)}$
- $f(x) = \frac{a(x-b)(x-c)}{(x-d)(x-e)}$ 2 zeros Touch

$\frac{-4(x+2)(x+2)}{(x+3)(x-1)}$

Long Division

So, $x^4 + 4x^3 + 55x^2 + 102x + 50 = (x+1)(x+1)(x^2 + 2x + 50)$.

We still need to write $x^2 + 2x + 50$ as a product of linear factors.

We can use the quadratic formula to find the zeros of $x^2 + 2x + 50 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(50)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{-196}}{2}$$
$$= \frac{-2 \pm 14i}{2}$$
$$= -1 \pm 7i$$

$(x+1)(x+1)(x+1-7i)(x+1+7i)$

Zeros $= (-1+7i)\ (-1-7i)$

Factors $(x-(-1+7i))(x-(-1-7i))$

$(x+1-7i)(x+1+7i)$

3, $1+i$, $1-i$

$(x-3)(x-1+i)(x-1-i)$

$((x-1)+i)((x-1)-i)$

$x^2-2x+1-i^2$

$x^2-2x+1+1$

$(x-3)(x^2-2x+2)$

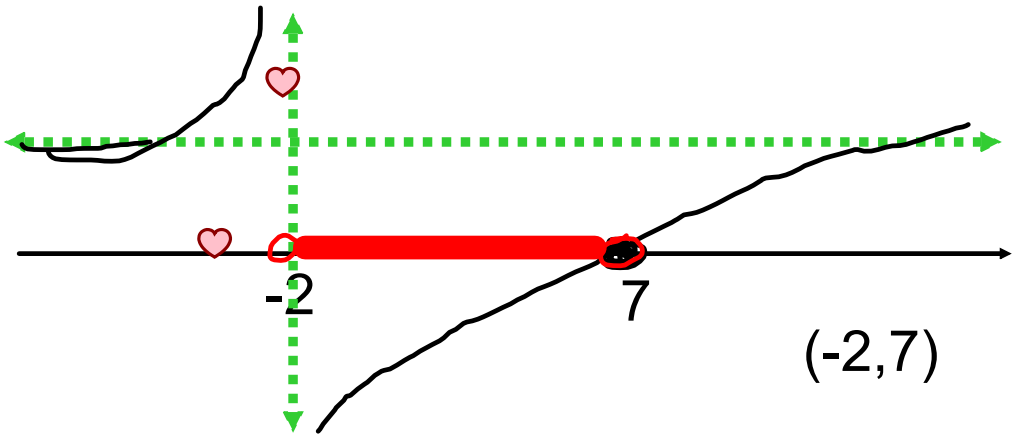
$$\begin{array}{r} x^3 - 2x^2 + 2x \\ -3x^2 + 6x - 6 \\ \hline x^3 - 5x^2 + 8x - 6 \end{array}$$

Solving a rational inequality: Problem type 1

Solve the following inequality.

$\frac{x-7}{x+2} < 0$ less than zero = under water

Write your answer using interval notation.



SPECIAL

If ZN = ZD what?????

$\frac{x+2}{x+2} = 1$ hole in the graph at x = -2