Mirrors and Lenses

Mirrors and lenses are able to present images of objects. We need to determine the location and nature of the images which are formed by any of these optical instruments or in combination of them.

The characteristics of the images such as location, orientation such us inverted, erect or right-left or left-right reversal, magnification, and description if the image is real or virtual.

Sign conventions have been established and it is important to understand the conventions for mirror, refracting surfaces, and lenses.

Mirrors:

A mirror has a front and a back.
The mirrors can be flat, concave, or convex

The distance from the object to the mirror = \( p \)
The height of the object = \( h \) is measured from the bottom of the object to the top.
A downward direction yields a negative value.

The distance from the image to the mirror = \( q \)
The height of the image = \( h' \) measured from the bottom of the image to the top.
A downward direction yields a negative value.

The magnification \( M \) of the optical instrument is defined as
\[
M = \frac{h'}{h} = -\left( \frac{q}{p} \right)
\]

The radius (\( R \)) is the radius of curvature of the surface of the mirror, and it is of importance.
The radius of curvature of a flat mirror is infinite.
The distance of \( R / 2 = f \) = the focal distance.
The focal \( F \) is located a distance \( f \) from to the surface of the mirror

When the object is located in front of the mirror the value of \( p \) has a + sign, the object is a real object.
While when the object is located in the back of the mirror \( p \) has a - sign, the object is virtual.

When the image is located in front of the mirror the image distance \( q \) has a + sign, the image is real.
When the image is located in the back of the mirror the distance \( q \) has a - sign, the image is virtual.

When \( R \), and \( f \) are in front of the mirror their values are + and the mirror is concave.
When \( R \), and \( f \) are in the back of the mirror their values are - and the mirror is convex.

When \( M \) is positive, implies the image is upright, that is erect
\( M = 1 \) implies no magnification and an upright image
When \( M \) is negative, implies the image is inverted which is the negative of upright.
\( M = -1 \) implies no magnification and an inverted image

The lens Equation:
\[
\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{f}
\]
**A Flat mirror:** $R = \infty \implies f = \infty \implies (2/R) = 0 = (1/f)$

$p = -q$

$h = h'$ and both are positive. $\implies M = 1$

As the object is in front, the image is behind the mirror, upright, and left-right reversal.

$M = 1$, the image is unmagnified, and virtual

**A Concave mirror:** $R$ and $f$ are $+$

When the object is located between the focal point and the center of curvature or beyond the center of curvature, into infinity, then:

- $p$ is $+$ Real object
- $q$ is $+$ Real image, inverted,

When $p > R$ then $M$ is negative and $> -1$

When $R > p > F$ then $M$ is negative and can be smaller than $-1$.

When $p = F$ then the image is located at infinity, that is one finds a ser of parallel rays.

When the object is located between the focal point and the surface of the mirror: $\ F > p >$ mirror surface

- $p$ is $+$ Real object
- $q$ is $-$ Virtual image, in the back of the mirror, upright, $M$ is $+$ and $M > 1$

**Convex Mirror:** $R$ and $f$ are $-$

- $p$ is $+$ Real object
- $q$ is $-$ Virtual image, in the back of the mirror, upright, $M$ is $+$ and $M < 1$

**Refracting Surfaces:**

Refracting surfaces implies the presence of two media with their corresponding index of refraction.

The equation is given by: \[ \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \]

Note: \[ M = \frac{\frac{h'}{h} = -\frac{n_1 q}{n_2 p}} \]

Consider the ray originating in medium 1 with index of refraction $n_1$ refracting into medium 2 with index of refraction $n_2$. Also consider the surface of medium 2 to be convex or concave.

Note: The front side of the surface is the side where the light is incident on, that is where the light source most likely is located.

The distance, $p$, from the object to the surface is $+$ when the object is in front of the surface, the object is then real.

The distance $p$ is $-$ when the object is in the back of the surface, the object is virtual.

The distance, $q$, from the image to the surface is $+$ when the image is in the back of surface, the image is real.

$R =$ the radius of curvature is $+$ when the center of curvature is in back of the surface.

$R =$ the radius of curvature is $-$ when the center of curvature is in front of the surface.
Medium 1 is more dense than medium and surface of medium 2 is convex; we have the effect of convergent rays into a point in medium 2 where the image is located.

\[ p \text{ corresponds to a real object, then } p \text{ is } + \]
\[ q \text{ corresponds to a real image, hence } q \text{ is } + \text{, located in medium 2.} \]
\[ R \text{ corresponds to the radius of curvature is } + \text{ located inside the surface.} \]

Medium 1 is less dense than medium 2, that is, \( n_1 > n_2 \) and the ray originates in medium 2. The surface of medium 1 is convex.

\[ p \text{ is located in medium 1, a real object, hence } p \text{ is } +. \]
\[ q \text{ is also located also in medium 1, hence it is a real image and } q \text{ is } -. \]
\[ R \text{ corresponds to the radius of curvature is } - \text{ and it corresponds to a surface of medium 1 which is concave.} \]

Note: If the surface is a flat surface, it means that the radius of curvature of the surface is infinite, hence the term \( (1/R) = 0 \). This important when solving a problem with this condition.

**Thin Lenses**

There are three rays that are used to form the ray diagram for a thin lens.

1. The ray is parallel to the optic axis. Upon refraction by the lens, the ray passes through (or appears to come from) on of the focal points.
2. The ray drawn passing through the center of the lens. This ray continues in a straight line.
3. The ray drawn through the fixed focal point F, this ray the emerges from the lens in a path that is parallel to the optic axis.

The equations are:

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]
\[ \frac{1}{f} = (n - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \]

**Known as the thin lens equations.**

**Lens makers’ equation**

Note: The direction of the ray determines the front and the back of the lens. The first surface in the path of the ray is the front of the lens.

The object distance \( p \) is + when the object is in front of the lens.
The object distance \( p \) is - when the object is in the back of the lens.

The image distance \( q \) is + when the image is in the back of the lens.
The image distance \( q \) is - when the image is in the front of the lens.

The radius of curvature \( R_1 \) and \( R_2 \) are positive (+) if the center of curvature is in the back of the lens.
The radius of curvature \( R_1 \) and \( R_2 \) are negative (-) if the center of curvature is in the front of the lens.
For a **double convex lens**: (Converging lens) $p > R_2$

- $p, q, f, R_1$, are positive.
- $R_2$ is negative
- The image is real and inverted. $M > 0$

For a double concave lens: (Diverging lens) $|p| > |f|$

- $p, R_2$, are positive.
- $q, f, R_1$, are negative.
- The image is virtual, erect and diminished $0 < M < 1$

For a double convex lens: (Converging lens) $p < f$

- $p, f, R_1$, are positive.
- $q, R_1$, are negative.
- The image is virtual, erect, and enlarged $M > 1$

**Aberrations:**

- **Spherical**  It is a variations in focal points for parallel incident rays.
- **Chromatic**  rays of light of different wavelength focus at different points when refracted by a lens.