Chapter 18

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The same potential difference exists across all elements connected in parallel with each other, while the current through each element is inversely proportional to the resistance of that element ($I = \Delta V/R$). Thus, both (b) and (c) are true statements while the other choices are false.

2. In a series connection, the same current exists in each element. The potential difference across a resistor in this series connection is directly proportional to the resistance of that resistor, $\Delta V = IR$, and independent of its location within the series connection. The only true statement among the listed choices is (c).

3. For these parallel resistors,

$$\frac{1}{R_{eq}} = \frac{1}{1.00\ \Omega} + \frac{1}{2.00\ \Omega} = \frac{2+1}{2.00\ \Omega} \text{ and } R_{eq} = \frac{2.00\ \Omega}{3} = 0.667\ \Omega$$

Choice (c) is the correct answer.

4. The total power dissipated is $P_{total} = P_1 + P_2 = 120\ \text{W} + 60.0\ \text{W} = 180\ \text{W}$, while the potential difference across this series combination is $\Delta V = 120\ \text{V}$. The current drawn through the series combination is then $I = \dfrac{P_{total}}{\Delta V} = \dfrac{180\ \text{W}}{120\ \text{V}} = 1.5\ \text{A}$, and (b) is the correct choice.

5. The equation of choice (b) is the result of a correct application of Kirchhoff's junction rule at either of the two junctions in the circuit. The equation of choice (c) results from a correct application of Kirchhoff's loop rule to the lower loop in the circuit, while the equation of choice (d) is obtained by correctly applying the loop rule to the loop forming the outer perimeter of the circuit. The equation of choice (a) is the result of an incorrect application (involving 2 sign errors) of the loop rule to the upper loop in the circuit. The correct answer is choice (a).

6. The equivalent resistance of the parallel combination consisting of the 4.0-Ω, 6.0-Ω, and 10-Ω resistors is $R_p = 1.9\ \Omega$. This resistance is in series with a 2.0-Ω resistor, making the total resistance of the circuit $R_{total} = 3.9\ \Omega$. The total current supplied by the battery is $I_{total} = \Delta V/R_{total} = 12\ \text{V}/3.9\ \Omega = 3.1\ \text{A}$. Thus, the potential difference across each resistor in the parallel combination is $\Delta V_p = R_p I_{total} = (1.9\ \Omega)(3.1\ \text{A}) = 5.9\ \text{V}$ and the current through the 10 Ω resistor is $I_{10} = \Delta V_p/10\ \Omega = 5.9\ \text{V}/10\ \Omega = 0.59\ \text{A}$. Choice (a) is the correct answer.

8. When the two identical resistors are in series, the current supplied by the battery is $I = \Delta V/2R$, and the total power delivered is $P_s = (\Delta V)I = (\Delta V)^2/2R$. With the resistors connected in parallel, the potential difference across each resistor is ΔV and the power delivered to each resistor is $P_1 = (\Delta V)^2/R$. Thus, the total power delivered in this case is

$$P_p = 2P_1 = 2\frac{(\Delta V)^2}{R} = 4\left[\frac{(\Delta V)^2}{2R}\right] = 4P_s = 4(8.0\ \text{W}) = 32\ \text{W}$$

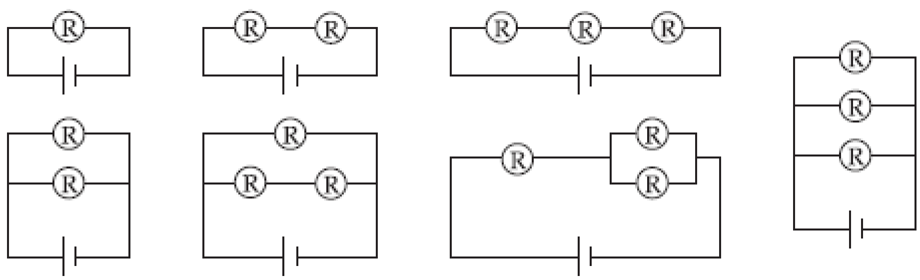
and (b) is the correct choice.

9. The equivalent resistance for the series combination of five identical resistors is $R_{eq} = 5R$, and the equivalent capacitance of five identical capacitors in parallel is $C_{eq} = 5C$. The time constant for the circuit is therefore $\tau = R_{eq}C_{eq} = (5R)(5C) = 25RC$ and (d) is the correct choice.

10. When the switch is closed, the current has a large initial value but decreases exponentially in time. The bulb will glow brightly at first, but fade rapidly as the capacitor charges. After a time equal to many time constants of the circuit, the current is essentially zero and the bulb does not glow. The correct answer is choice (c).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2.



4. A short circuit can develop when the last bit of insulation frays away between the two conductors in a lamp cord. Then the two conductors touch each other, creating a low resistance path in parallel with the lamp. The lamp will immediately go out, carrying no current and presenting no danger. A very large current will be produced in the power source, the house wiring, and the wire in the lamp cord up to and through the short. The circuit breaker will interrupt the circuit quickly but not before considerable heating and sparking is produced in the short-circuit path.

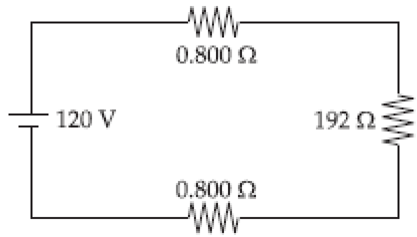
6. A wire or cable in a transmission line is thick and made of material with very low resistivity. Only when its length is very large does its resistance become significant. To transmit power over a long distance it is most efficient to use low current at high voltage. The power loss per unit length of the transmission line is $P_{loss}/L = I^2(R/L)$, where R/L is the resistance per unit length of the line. Thus, a low current is clearly desirable, but to transmit a significant amount of power $P = (\Delta V)I$ with low current, a high voltage must be used.

PROBLEM SOLUTIONS

18.1 From $\Delta V = I(R + r)$, the internal resistance is

$$r = \frac{\Delta V}{I} - R = \frac{9.00 \text{ V}}{0.117 \text{ A}} - 72.0\ \Omega = \boxed{4.92\ \Omega}$$

18.3 (a) The bulb acts as a 192-Ω resistor (see below), so the circuit diagram is:



(b) For the bulb in use as intended, $R_{bulb} = (\Delta V)^2/P = (120\text{ V})^2/75.0\text{ W} = 192\ \Omega$.

Now, assuming the bulb resistance is unchanged, the current in the circuit shown is

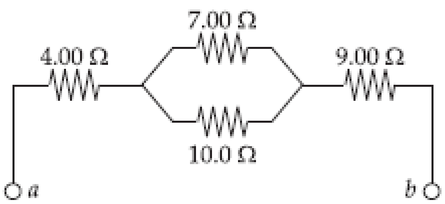
$$I = \frac{\Delta V}{R_{eq}} = \frac{120\text{ V}}{0.800\ \Omega + 192\ \Omega + 0.800\ \Omega} = 0.620\text{ A}$$

and the actual power dissipated in the bulb is

$$P = I^2 R_{bulb} = (0.620\text{ A})^2 (192\ \Omega) = \boxed{73.8\text{ W}}$$

18.5 (a) The equivalent resistance of the two parallel resistors is

$$R_p = \left(\frac{1}{7.00\ \Omega} + \frac{1}{10.0\ \Omega}\right)^{-1} = 4.12\ \Omega$$



Thus,

$$R_{ab} = R_4 + R_p + R_9 = (4.00 + 4.12 + 9.00)\ \Omega = \boxed{17.1\ \Omega}$$

18.7 When connected in series, we have $R_1 + R_2 = 690\ \Omega$ **[1]**

which we may rewrite as $R_2 = 690\ \Omega - R_1$ **[1a]**

When in parallel, $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{150\ \Omega}$ or $\frac{R_1 R_2}{R_1 + R_2} = 150\ \Omega$ **[2]**

Substitute Equations [1] and [1a] into Equation [2] to obtain:

$$\frac{R_1(690\ \Omega - R_1)}{690\ \Omega} = 150\ \Omega \quad \text{or} \quad R_1^2 - (690\ \Omega)R_1 + (690\ \Omega)(150\ \Omega) = 0$$ **[3]**

Using the quadratic formula to solve Equation [3] gives

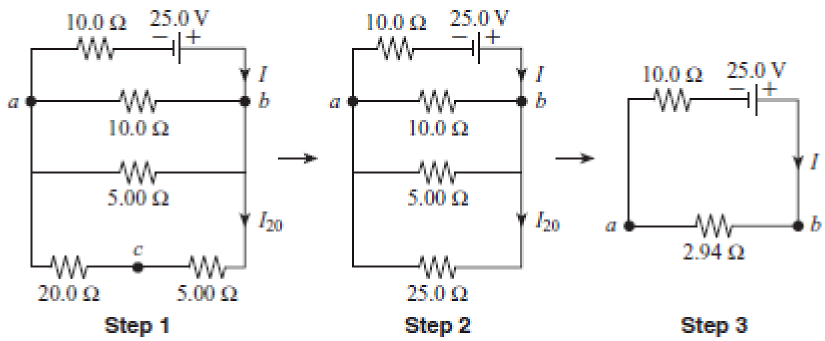
$$R_1 = \frac{690\ \Omega \pm \sqrt{(690\ \Omega)^2 - 4(690\ \Omega)(150\ \Omega)}}{2}$$

with two solutions of $R_1 = 470\ \Omega$ and $R_1 = 220\ \Omega$

Then Equation [1a] yields $R_2 = 220\ \Omega$ or $R_2 = 470\ \Omega$

Thus, the two resistors have resistances of $\boxed{220\ \Omega \text{ and } 470\ \Omega}$.

18.9 (a) Using the rules for combining resistors in series and parallel, the circuit reduces as shown below:



From the figure of Step 3, observe that

$$I = \frac{25.0\text{ V}}{10.0\ \Omega + 2.94\ \Omega} = 1.93\text{ A} \quad \text{and} \quad \Delta V_{ab} = I(2.94\ \Omega) = (1.93\text{ A})(2.94\ \Omega) = \boxed{5.67\text{ V}}.$$

(b) From the figure of Step 1, observe that $I_{20} = \dfrac{\Delta V_{ab}}{25.0\ \Omega} = \dfrac{5.67\text{ V}}{25.0\ \Omega} = \boxed{0.227\text{ A}}.$

18.11 The equivalent resistance is $R_{eq} = R + R_p$, where R_p is the total resistance of the three parallel branches:

$$R_p = \left(\frac{1}{120\ \Omega} + \frac{1}{40\ \Omega} + \frac{1}{R + 5.0\ \Omega}\right)^{-1} = \left(\frac{1}{30\ \Omega} + \frac{1}{R + 5.0\ \Omega}\right)^{-1} = \frac{(30\ \Omega)(R + 5.0\ \Omega)}{R + 35\ \Omega}.$$

Thus, $$75\ \Omega = R + \frac{(30\ \Omega)(R + 5.0\ \Omega)}{R + 35\ \Omega} = \frac{R^2 + (65\ \Omega)R + 150\ \Omega^2}{R + 35\ \Omega}.$$

which reduces to $R^2 - (10\ \Omega)R - 2\,475\ \Omega^2 = 0$ or $(R - 55\ \Omega)(R + 45\ \Omega) = 0.$

Only the positive solution is physically acceptable, so $R = \boxed{55\ \Omega}.$

18.13 The resistors in the circuit can be combined in the stages shown below to yield an equivalent resistance of $R_{ad} = (63/11)\ \Omega$.

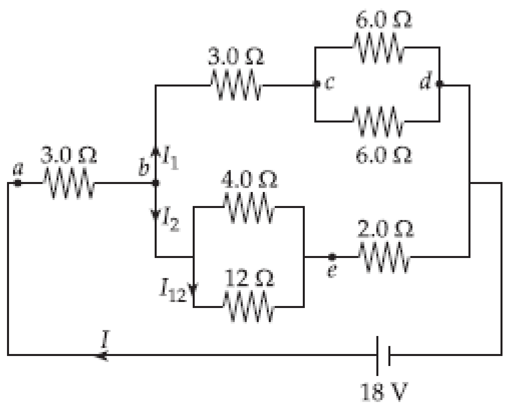


Figure 1

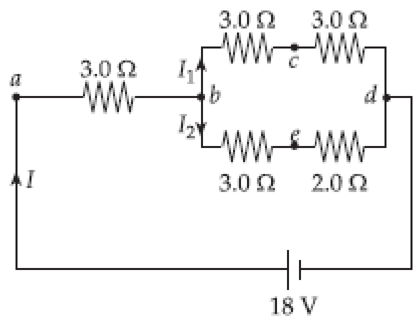


Figure 2

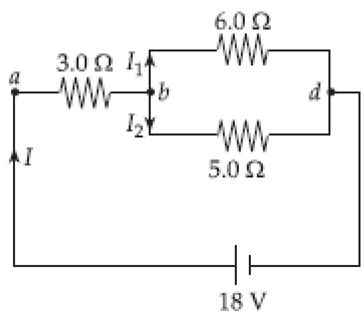


Figure 3

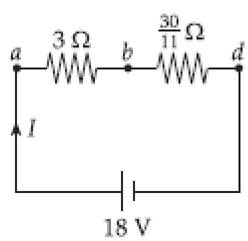


Figure 4

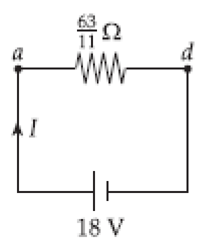


Figure 5

From Figure 5, $I = \dfrac{(\Delta V)_{ad}}{R_{ad}} = \dfrac{18\text{ V}}{(63/11)\ \Omega} = 3.1\text{ A}$

Then, from Figure 4, $(\Delta V)_{bd} = I\,R_{bd} = (3.1\text{ A})(30/11\ \Omega) = 8.5\text{ V}$

Now, look at Figure 2 and observe that

$$I_2 = \frac{(\Delta V)_{bd}}{3.0\ \Omega + 2.0\ \Omega} = \frac{8.5\text{ V}}{5.0\ \Omega} = 1.7\text{ A}$$

so $(\Delta V)_{be} = I_2 R_{be} = (1.7\text{ A})(3.0\ \Omega) = 5.1\text{ V}$

Finally, from Figure 1, $I_{12} = \dfrac{(\Delta V)_{be}}{R_{12}} = \dfrac{5.1\text{ V}}{12\ \Omega} = \boxed{0.43\text{ A}}$

18.31 (a) The time constant is: $\tau = RC = (75.0\times10^{3}\ \Omega)(25.0\times10^{-6}\text{ F}) = \boxed{1.88\text{ s}}$.

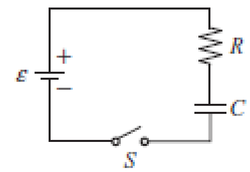
(b) At $t = \tau$, $q = 0.632Q_{\max} = 0.632(C\varepsilon) = 0.632(25.0\times10^{-6}\text{ F})(12.0\text{ V}) = \boxed{1.90\times10^{-4}\text{ C}}$.

18.33 (a) The time constant of an *RC* circuit is $\tau = RC$. Thus,

$$\tau = \left(1.00\times10^{6}\ \Omega\right)\left(5.00\times10^{-6}\ \text{F}\right) = \boxed{5.00\ \text{s}}$$

(b) $Q_{max} = C\varepsilon = \left(5.00\ \mu\text{F}\right)\left(30.0\ \text{V}\right) = \boxed{150\ \mu\text{C}}$

(c) To obtain the current through the resistor at time *t* after the switch is closed, recall that the charge on the capacitor at that time is $q = C\varepsilon\left(1 - e^{-t/\tau}\right)$ and the potential difference across a capacitor is $V_c = q/C$. Thus,



$$V_C = \frac{C\varepsilon\left(1 - e^{-t/\tau}\right)}{C} = \varepsilon\left(1 - e^{-t/\tau}\right)$$

Then, considering switch *S* to have been closed at time $t = 0$, apply Kirchhoff's loop rule around the circuit shown above to obtain

$$+\varepsilon - iR - V_C = 0 \qquad \text{or} \qquad i = \frac{\varepsilon - \varepsilon\left(1 - e^{-t/\tau}\right)}{R}$$

The current in the circuit at time *t* after the switch is closed is then $i = \left(\varepsilon/R\right)e^{-t/\tau}$, so the current in the resistor at $t = 10.0$ s is

$$i = \left(\frac{30.0\ \text{V}}{1.00\times10^{6}\ \Omega}\right)e^{-\frac{10.0\ \text{s}}{5.00\ \text{s}}} = \left(30.0\ \mu\text{A}\right)e^{-2.00} = \boxed{4.06\ \mu\text{A}}$$