Chapter 16

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The change in the potential energy of the proton is equal to the negative of the work done on it by the electric field. Thus,

$$\Delta PE = -W = -qE_x(\Delta x) = -\left(+1.6\times10^{-19}\text{ C}\right)\left(850\text{ N/C}\right)\left(2.5\text{ m}-0\right) = -3.4\times10^{-16}\text{ J}$$

and (b) is the correct choice for this question.

2. Because electric forces are conservative, the kinetic energy gained is equal to the decrease in electrical potential energy, or

$$KE = -PE = -q(\Delta V) = -(-1\text{ e})\left(+1.00\times10^{4}\text{ V}\right) = +1.00\times10^{4}\text{ eV}$$

so the correct choice is (a).

3. In a uniform electric field, the change in electric potential is $\Delta V = -E_x(\Delta x)$, giving

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{\left(V_f - V_i\right)}{\left(x_f - x_i\right)} = -\frac{(190\text{ V}-120\text{ V})}{(5.0\text{ m}-3.0\text{ m})} = -35\text{ V/m} = -35\text{ N/C}$$

and it is seen that the correct choice is (d).

4. From conservation of energy, $KE_f + PE_f = KE_i + PE_i$, or $\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + qV_A - qV_B$, the final speed of the nucleus is

$$v_B = \sqrt{v_A^2 + \frac{2q(V_A - V_B)}{m}}$$

$$= \sqrt{\left(6.20\times10^{5}\text{ m/s}\right)^2 + \frac{2\left[2\left(1.60\times10^{-19}\text{ C}\right)(1.50-4.00)\times10^{3}\text{ V}\right]}{6.63\times10^{-27}\text{ kg}}} = 3.78\times10^{5}\text{ m/s}$$

Thus, the correct answer is choice (b).

7. With the given specifications, the capacitance of this parallel-plate capacitor will be

$$C = \frac{\kappa\,\epsilon_0\,A}{d} = \frac{\left(1.00\times10^{2}\right)\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(1.00\text{ cm}^2\right)}{1.00\times10^{-3}\text{ m}}\left(\frac{1\text{ m}^2}{10^{4}\text{ cm}^2}\right)$$

$$= 8.85\times10^{-11}\text{ F} = 88.5\times10^{-12}\text{ F} = 88.5\text{ pF}$$

and the correct choice is (a).

11. Capacitances connected in parallel all have the same potential difference across them and the equivalent capacitance, $C_{eq} = C_1 + C_2 + C_3 + \ldots$, is larger than the capacitance of any one of the capacitors in the combination. Thus, choice (c) is a true statement. The charge on a capacitor is $Q = C(\Delta V)$, so with ΔV constant, but the capacitances different, the capacitors all store different charges that are proportional to the capacitances, making choices (a), (b), (d), and (e) all false. Therefore, (c) is the only correct answer.

12. For a series combination of capacitors, the magnitude of the charge is the same on all plates of capacitors in the combination. Also, the equivalent capacitance is always less than any individual capacitance in the combination. Therefore, choice (a) is true while choices (b) and (c) are both false. The potential difference across a capacitor is $\Delta V = Q/C$, so with Q constant, capacitors having different capacitances will have different potential differences across them, with the largest potential difference being across the capacitor with the smallest capacitance. This means that choices (d) and (e) are false, and choice (f) is true. Thus, both choices (a) and (f) are true statements.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

4. To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

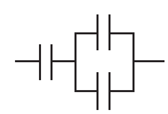
8. There are eight different combinations that use all three capacitors in the circuit. These combinations and their equivalent capacitances are:

All three capacitors in series: $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$

All three capacitors in parallel: $C_{eq} = C_1 + C_2 + C_3$

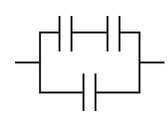
One capacitor in series with a parallel combination of the other two:

$$C_{eq} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_3}\right)^{-1},\ C_{eq} = \left(\frac{1}{C_3 + C_1} + \frac{1}{C_2}\right)^{-1},\ C_{eq} = \left(\frac{1}{C_2 + C_3} + \frac{1}{C_1}\right)^{-1}$$



One capacitor in parallel with a series combination of the other two:

$$C_{eq} = \left(\frac{C_1 C_2}{C_1 + C_2}\right) + C_3,\ C_{eq} = \left(\frac{C_3 C_1}{C_3 + C_1}\right) + C_2,\ C_{eq} = \left(\frac{C_2 C_3}{C_2 + C_3}\right) + C_1$$



PROBLEM SOLUTIONS

16.1 (a) Because the electron has a negative charge, it experiences a force in the direction opposite to the field and, when released from rest, will move in the negative *x*-direction. The work done on the electron by the field is

$$W = F_x(\Delta x) = (qE_x)\Delta x = (-1.60\times10^{-19}\text{ C})(375\text{ N/C})(-3.20\times10^{-2}\text{ m})$$
$$= \boxed{1.92\times10^{-18}\text{ J}}$$

(b) The change in the electric potential energy is the negative of the work done on the particle by the field. Thus,

$$\Delta PE = -W = \boxed{-1.92\times10^{-18}\text{ J}}$$

(c) Since the Coulomb force is a conservative force, conservation of energy gives $\Delta KE + \Delta PE = 0$, or $KE_f = \frac{1}{2}m_e v_f^2 = KE_i - \Delta PE = 0 - \Delta PE$, and

$$v_f = \sqrt{\frac{-2(\Delta PE)}{m_e}} = \sqrt{\frac{-2(-1.92\times10^{-18}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = \boxed{2.05\times10^{6}\text{ m/s in the }-x\text{-direction}}$$

16.3 The work done by the agent moving the charge out of the cell is

$$W_{\text{input}} = -W_{\text{field}} = -(-\Delta PE_e) = +q(\Delta V)$$
$$= (1.60\times10^{-19}\text{ C})(+90\times10^{-3}\text{ J/C}) = \boxed{1.4\times10^{-20}\text{ J}}$$

16.4 Assuming the sphere is isolated, the excess charge on it is uniformly distributed over its surface. Under this spherical symmetry, the electric field outside the sphere is the same as if all the excess charge on the sphere were concentrated as a point charge located at the center of the sphere.

Thus, at $r = 8.00$ cm $> R_{\text{sphere}} = 5.00$ cm, the electric field is $E = k_e Q/r^2$. The required charge then has magnitude $|Q| = Er^2/k_e$, and the number of electrons needed is

$$n = \frac{|Q|}{e} = \frac{Er^2}{k_e e} = \frac{(1.50\times10^{5}\text{ N/C})(8.00\times10^{-2}\text{ m})^2}{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})} = \boxed{6.67\times10^{11}\text{ electrons}}$$

16.12 (a) $$V_A = \sum_i \frac{k_e q_i}{r_i} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-15.0\times10^{-9}\text{ C}}{2.00\times10^{-2}\text{ m}} + \frac{27.0\times10^{-9}\text{ C}}{2.00\times10^{-2}\text{ m}}\right) = \boxed{+5.39\text{ kV}}$$

(b) $$V_B = \sum_i \frac{k_e q_i}{r_i} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-15.0\times10^{-9}\text{ C}}{1.00\times10^{-2}\text{ m}} + \frac{27.0\times10^{-9}\text{ C}}{1.00\times10^{-2}\text{ m}}\right) = \boxed{+10.8\text{ kV}}$$

16.26 (a) $$C = \frac{Q}{\Delta V} = \frac{27.0\ \mu\text{C}}{9.00\text{ V}} = \boxed{3.00\ \mu\text{F}}$$

(b) $$Q = C(\Delta V) = (3.00\ \mu\text{F})(12.0\text{ V}) = \boxed{36.0\ \mu\text{C}}$$

16.28 (a) $$C = \frac{Q}{V} = \frac{10.0\ \mu\text{C}}{10.0\text{ V}} = \boxed{1.00\ \mu\text{F}}$$

(b) $$V = \frac{Q}{C} = \frac{100\ \mu\text{C}}{1.00\ \mu\text{F}} = \boxed{100\text{ V}}$$

16.33 (a) Capacitors in a series combination store the same charge, $Q = C_{eq}(\Delta V)$, where C_{eq} is the equivalent capacitance and ΔV is the potential difference maintained across the series combination. The equivalent capacitance for the given series combination is $1/C_{eq} = 1/C_1 + 1/C_2$, or $C_{eq} = C_1C_2/(C_1 + C_2)$, giving

$$C_{eq} = \frac{(2.50\ \mu\text{F})(6.25\ \mu\text{F})}{2.50\ \mu\text{F} + 6.25\ \mu\text{F}} = 1.79\ \mu\text{F}$$

and the charge stored on each capacitor in the series combination is

$$Q = C_{eq}(\Delta V) = (1.79\ \mu\text{F})(6.00\ \text{V}) = \boxed{10.7\ \mu\text{C}}$$

(b) When connected in parallel, each capacitor has the same potential difference, $\Delta V = 6.00$ V, maintained across it. The charge stored on each capacitor is then

For $C_1 = 2.50\ \mu$F: $Q_1 = C_1(\Delta V) = (2.50\ \mu\text{F})(6.00\ \text{V}) = \boxed{15.0\ \mu\text{C}}$

For $C_2 = 6.25\ \mu$F: $Q_2 = C_2(\Delta V) = (6.25\ \mu\text{F})(6.00\ \text{V}) = \boxed{37.5\ \mu\text{C}}$

16.34 (a) $C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$

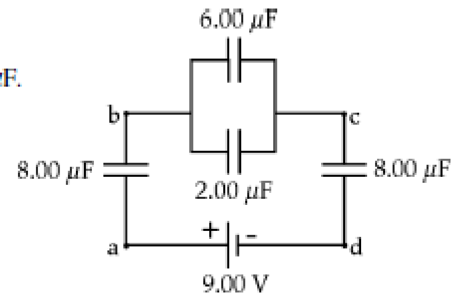
(b) In a parallel combination, the full potential difference maintained between the terminals of the battery exists across each capacitor. Thus,

$$\Delta V_1 = \Delta V_2 = \Delta V_{battery} = \boxed{9.00\ \text{V}}$$

(c) $Q_1 = C_1(\Delta V_1) = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V_2) = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

16.35 (a) First, we replace the parallel combination between points b and c by its equivalent capacitance, $C_{bc} = 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = 8.00\ \mu\text{F}$. Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore



$$\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00\ \mu\text{F}}$$

giving $C_{eq} = \dfrac{8.00\ \mu\text{F}}{3} = \boxed{2.67\ \mu\text{F}}$

(b) The charge stored on each capacitor in the series combination is

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}(\Delta V_{ad}) = (2.67\ \mu\text{F})(9.00\ \text{V}) = 24.0\ \mu\text{C}$$

Then, note that $\Delta V_{bc} = Q_{bc}/C_{bc} = 24.0\ \mu\text{C}/8.00\ \mu\text{F} = 3.00\ \text{V}$. The charge on each capacitor in the original circuit is:

On the 8.00 μF between a and b: $Q_8 = Q_{ab} = \boxed{24.0\ \mu\text{C}}$

On the 8.00 μF between c and d: $Q_8 = Q_{cd} = \boxed{24.0\ \mu\text{C}}$

On the 2.00 μF between b and c: $Q_2 = C_2(\Delta V_{bc}) = (2.00\ \mu\text{F})(3.00\ \text{V}) = \boxed{6.00\ \mu\text{C}}$

On the 6.00 μF between b and c: $Q_6 = C_6(\Delta V_{bc}) = (6.00\ \mu\text{F})(3.00\ \text{V}) = \boxed{18.0\ \mu\text{C}}$

(c) Note that $\Delta V_{ab} = Q_{ab}/C_{ab} = 24.0\ \mu\text{C}/8.00\ \mu\text{F} = 3.00\ \text{V}$, and that $\Delta V_{cd} = Q_{cd}/C_{cd} = 24.0\ \mu\text{C}/8.00\ \mu\text{F} = 3.00\ \text{V}$. We earlier found that $\Delta V_{bc} = 3.00\ \text{V}$, so we conclude that the potential difference across each capacitor in the circuit is

$$\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\ \text{V}}$$

16.45 Energy stored $= \dfrac{Q^2}{2C} = \dfrac{1}{2}C(\Delta V)^2 = \dfrac{1}{2}(4.50\times10^{-6}\ \text{F})(12.0\ \text{V})^2 = \boxed{3.24\times10^{-4}\ \text{J}}$

16.48 The energy transferred to the water is

$$W = \frac{1}{100}\left[\frac{1}{2}Q(\Delta V)\right] = \frac{(50.0\ \text{C})(1.00\times10^8\ \text{V})}{200} = 2.50\times10^7\ \text{J}$$

Thus, if m is the mass of water boiled away, $W = m\left[c(\Delta T) + L_v\right]$ becomes

$$2.50\times10^7\ \text{J} = m\left[\left(4186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(100^\circ\text{C} - 30.0^\circ\text{C}) + 2.26\times10^6\ \text{J/kg}\right]$$

giving $m = \dfrac{2.50\times10^7\ \text{J}}{\left[2.93\times10^5\ \text{J/kg} + 2.26\times10^6\ \text{J/kg}\right]} = \boxed{9.79\ \text{kg}}$