Chapter 24

2. The magnitude is $\Delta U = e\Delta V = 1.2 \times 10^9$ eV = 1.2 GeV.

5. **THINK** The electric field produced by an infinite sheet of charge is normal to the sheet and is uniform.

EXPRESS The magnitude of the electric field produced by the infinite sheet of charge is $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by

$$\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x.$$

ANALYZE Thus, for $\sigma = 0.10\times10^{-6}\ \text{C/m}^2$ and $\Delta V = 50\ \text{V}$, we have

$$\Delta x = \frac{2\varepsilon_0\Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(50\ \text{V})}{0.10\times10^{-6}\ \text{C/m}^2} = 8.8\times10^{-3}\,\text{m}.$$

LEARN Equipotential surfaces are always perpendicular to the electric field lines. Figure 24-5(a) depicts the electric field lines and equipotential surfaces for a uniform electric field.

7. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 24-18: $\int \vec{E}\cdot d\vec{s} = 0$). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E}\cdot d\vec{s} = -4.00\int_0^4 x\,dx = -4.00\left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32.0$ V.

10. In the "inside" region between the plates, the individual fields (given by Eq. 24-13) are in the same direction ($-\hat{\text{i}}$):

$$\vec{E}_{\text{in}} = -\left(\frac{50\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} + \frac{25\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\right)\hat{\text{i}} = -(4.2\times10^3\ \text{N/C})\hat{\text{i}}\,.$$

In the "outside" region where $x > 0.5$ m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} + \frac{25\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} = -(1.4\times10^3\ \text{N/C})\hat{\text{i}}\,.$$

Therefore, by Eq. 24-18, we have

$$\begin{aligned}\Delta V &= -\int_0^{0.8} \vec{E}\cdot d\vec{s} = -\int_0^{0.5} \left|\vec{E}_{\text{in}}\right| dx - \int_{0.5}^{0.8} \left|\vec{E}_{\text{out}}\right| dx = -\left(4.2\times10^3\right)\left(0.5\right) - \left(1.4\times10^3\right)\left(0.3\right) \\ &= 2.5\times10^3\ \text{V}.\end{aligned}$$

13. (a) The charge on the sphere is

$$q = 4\pi\varepsilon_0 VR = \frac{(200\ \text{V})(0.15\ \text{m})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 3.3\times10^{-9}\ \text{C}.$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma = \frac{q}{4\pi R^2} = \frac{3.3\times10^{-9}\ \text{C}}{4\pi\left(0.15\ \text{m}\right)^2} = 1.2\times10^{-8}\ \text{C/m}^2.$$

21. We use Eq. 24-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.47\times3.34\times10^{-30}\ \text{C}\cdot\text{m}\right)}{\left(52.0\times10^{-9}\ \text{m}\right)^2} = 1.63\times10^{-5}\ \text{V}.$$

35. We use Eq. 24-41:

$$\begin{aligned}E_x(x,y) &= -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = -2(2.0\text{V}/\text{m}^2)x;\\ E_y(x,y) &= -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = 2(3.0\text{V}/\text{m}^2)y\,.\end{aligned}$$

We evaluate at $x = 3.0$ m and $y = 2.0$ m to obtain

$$\vec{E} = (-12\ \text{V/m})\hat{\text{i}} + (12\ \text{V/m})\hat{\text{j}}\,.$$

47. The *escape speed* may be calculated from the requirement that the initial kinetic energy (of *launch*) be equal to the absolute value of the initial potential energy (compare with the gravitational case in Chapter 14). Thus,

$$\frac{1}{2}mv^2 = \frac{eq}{4\pi\varepsilon_0 r}$$

where m = 9.11 × 10^{-31} kg, e = 1.60 × 10^{-19} C, q = 10000e, and r = 0.010 m. This yields v = 22490 m/s $\approx 2.2\times10^4$ m/s.

68. The potential energy of the two-charge system is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\,\text{C})(-4.00\times10^{-6}\,\text{C})}{\sqrt{(3.50+2.00)^2 + (0.500-1.50)^2}\ \text{cm}}$$
$$= -1.93\ \text{J}.$$

Thus, –1.93 J of work is needed.