Chapter 12

6. Let $\ell_1 = 1.5\,\text{m}$ and $\ell_2 = (5.0 - 1.5)\ \text{m} = 3.5\ \text{m}$. We denote tension in the cable closer to the window as F_1 and that in the other cable as F_2. The force of gravity on the scaffold itself (of magnitude $m_s g$) is at its midpoint, $\ell_3 = 2.5\,\text{m}$ from either end.

(a) Taking torques about the end of the plank farthest from the window washer, we find

$$F_1 = \frac{m_w g \ell_2 + m_s g \ell_3}{\ell_1 + \ell_2} = \frac{(80\,\text{kg})(9.8\,\text{m/s}^2)(3.5\,\text{m}) + (60\,\text{kg})(9.8\,\text{m/s}^2)(2.5\,\text{m})}{5.0\,\text{m}}$$
$$= 8.4 \times 10^2\ \text{N}.$$

(b) Equilibrium of forces leads to

$$F_1 + F_2 = m_s g + m_w g = (60\,\text{kg} + 80\,\text{kg})(9.8\,\text{m/s}^2) = 1.4 \times 10^3\ \text{N}$$

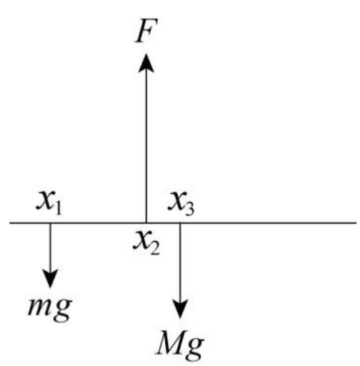
which (using our result from part (a)) yields $F_2 = 5.3 \times 10^2\,\text{N}$.

8. From $\vec{\tau} = \vec{r} \times \vec{F}$, we note that persons 1 through 4 exert torques pointing out of the page (relative to the fulcrum), and persons 5 through 8 exert torques pointing into the page.

(a) Among persons 1 through 4, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 2.

(b) Among persons 5 through 8, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 7.

9. The x axis is along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown on the diagram below. The nickels are at $x = x_1 = 0.120$ m, and m is their total mass.



The knife edge is at $x = x_2 = 0.455$ m and exerts force $\vec{F}$. The mass of the meter stick is M, and the force of gravity acts at the center of the stick, $x = x_3 = 0.500$ m. Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish:

$$Mg(x_3 - x_2) - mg(x_2 - x_1) = 0.$$

Thus,

$$M = \frac{x_2 - x_1}{x_3 - x_2} m = \left(\frac{0.455\,\text{m} - 0.120\,\text{m}}{0.500\,\text{m} - 0.455\,\text{m}}\right)(10.0\text{g}) = 74.4\ \text{g}.$$

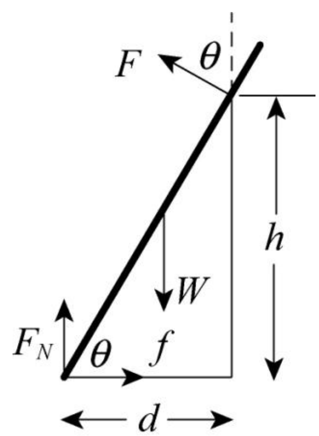
17. (a) With the pivot at the hinge, Eq. 12-9 gives

$$TL\cos\theta - mg\,\tfrac{L}{2} = 0\ .$$

This leads to $\theta = 78°$. Then the geometric relation $\tan\theta = L/D$ gives $D = 0.64$ m.

(b) A higher (steeper) slope for the cable results in a smaller tension. Thus, making D greater than the value of part (a) should prevent rupture.

37. The free-body diagram below shows the forces acting on the plank. Since the roller is frictionless, the force it exerts is normal to the plank and makes the angle θ with the vertical.



Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. F_N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$.

The equations of equilibrium are:

horizontal force components: $F\sin\theta - f = 0$

vertical force components: $F\cos\theta - W + F_N = 0$

torques: $F_N d - fh - W\ d - \tfrac{L}{2}\cos\theta\ = 0.$

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When $\theta = 70^\circ$ the plank just begins to slip and $f = \mu_s F_N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute F_N and f for $\theta = 70^\circ$, then use $\mu_s = f/F_N$ to compute the coefficient of friction.

The second equation gives $F = (W - F_N)/\cos\theta$ and this is substituted into the first to obtain

$$f = (W - F_N)\sin\theta/\cos\theta = (W - F_N)\tan\theta.$$

This is substituted into the third equation and the result is solved for F_N:

$$F_N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta}W = \frac{h(1+\tan^2\theta) - (L/2)\sin\theta}{h(1+\tan^2\theta)}W,$$

where we have used $d = h/\tan\theta$ and multiplied both numerator and denominator by $\tan\theta$. We use the trigonometric identity $1 + \tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$F_N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right).$$

Now we use this expression for F_N in $f = (W - F_N)\tan\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta.$$

Substituting these expressions for f and F_N into $\mu_s = f/F_N$ leads to

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}.$$

Evaluating this expression for $\theta = 70^\circ$, $L = 6.10$ m and $h = 3.05$ m gives

$$\mu_s = \frac{(6.1\,\text{m})\sin^2 70^\circ\cos 70^\circ}{2(3.05\,\text{m}) - (6.1\,\text{m})\sin 70^\circ\cos^2 70^\circ} = 0.34.$$