2. (a) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length $L$ of the rod, so the work done by the force of gravity is

$$ W = mgL = (0.341 \text{ kg})(9.80 \text{ m/s}^2)(0.452 \text{ m}) = 1.51 \text{ J}. $$

(b) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to $L$, but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is

$$ W = -mgL = -(0.341 \text{ kg})(9.80 \text{ m/s}^2)(0.452 \text{ m}) = -1.51 \text{ J}. $$

(c) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement.

(d) The force of gravity is conservative. The change in the gravitational potential energy of the ball-Earth system is the negative of the work done by gravity:

$$ \Delta U = -mgL = -(0.341 \text{ kg})(9.80 \text{ m/s}^2)(0.452 \text{ m}) = -1.51 \text{ J} $$
as the ball goes to the lowest point.

(e) Continuing this line of reasoning, we find

$$ \Delta U = +mgL = (0.341 \text{ kg})(9.80 \text{ m/s}^2)(0.452 \text{ m}) = 1.51 \text{ J} $$
as it goes to the highest point.

(f) Continuing this line of reasoning, we have $\Delta U = 0$ as it goes to the point at the same height.

(g) The change in the gravitational potential energy depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the potential energy is the same since the initial and final positions are the same.

15. We take the reference point for gravitational potential energy at the position of the marble when the spring is compressed.

(a) The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$, where $h = 20 \text{ m}$ is the height of the highest point. Thus,
\[ U_g = \left( 5.0 \times 10^{-3} \text{ kg} \right) \left( 9.8 \text{ m/s}^2 \right) (20 \text{ m}) = 0.98 \text{ J}. \]

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies \( \Delta U_g + \Delta U_s = 0 \), where \( \Delta U_g \) is the change in the spring’s elastic potential energy. Therefore, \( \Delta U_s = -\Delta U_g = -0.98 \text{ J} \).

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is \( U_s = 0.98 \text{ J} \). This must be \( \frac{1}{2} kx^2 \), where \( k \) is the spring constant and \( x \) is the initial compression. Consequently,

\[
k = \frac{2U_s}{x^2} = \frac{2(0.98 \text{ J})}{(0.080 \text{ m})^2} = 3.1 \times 10^2 \text{ N/m} = 3.1 \text{ N/cm}.
\]

23. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this \( t = 6.0 \text{ s} \) flight.

\[ \Delta y = v_y t - \frac{1}{2} gt^2 \]

This leads to \( \Delta y = -32 \text{ m} \). Therefore \( \Delta U = mg\Delta y = -318 \text{ J} = -3.2 \times 10^{-2} \text{ J} \).

25. (a) To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine — if it didn’t break — would have the greatest tension. Choosing upward positive, Newton’s second law leads to

\[
T - mg = m \frac{v^2}{r}
\]

where \( r = 18.0 \text{ m} \) and \( m = \frac{W}{g} = 688/9.8 = 70.2 \text{ kg} \). We find the \( v^2 \) from energy conservation (where the reference position for the potential energy is at the lowest point).

\[
mgh = \frac{1}{2} mv^2 \quad \Rightarrow \quad v^2 = 2gh
\]

where \( h = 3.20 \text{ m} \). Combining these results, we have

\[
T = mg + m\frac{2gh}{r} = mg \left( 1 + \frac{2h}{r} \right)
\]

which yields 933 N. Thus, the vine does not break.

(b) Rounding to an appropriate number of significant figures, we see the maximum tension is roughly \( 9.3 \times 10^2 \text{ N} \).
36. Let $F_N$ be the normal force of the ice on him and $m$ is his mass. The net inward force is $mg \cos \theta - F_N$ and, according to Newton’s second law, this must be equal to $mv^2/R$, where $v$ is the speed of the boy. At the point where the boy leaves the ice $F_N = 0$, so $g \cos \theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is

$$U = -mgR(1 - \cos \theta).$$

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives

$$0 = \frac{1}{2}mv^2 - mgR(1 - \cos \theta),$$

or $v^2 = 2gR(1 - \cos \theta)$. We substitute this expression into the equation developed from the second law to obtain $g \cos \theta = 2g(1 - \cos \theta)$. This gives $\cos \theta = 2/3$. The height of the boy above the bottom of the mound is

$$h = R \cos \theta = 2R/3 = 2(13.8 \text{ m})/3 = 9.20 \text{ m}.$$

66. (a) Since the speed of the crate of mass $m$ increases from 0 to 1.20 m/s relative to the factory ground, the kinetic energy supplied to it is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(300 \text{ kg})(120 \text{ m/s})^2 = 216 \text{ J}.$$

(b) The magnitude of the kinetic frictional force is

$$f = \mu F_N = \mu mg = (0.400)(300 \text{ kg})(9.8 \text{ m/s}^2) = 1.18 \times 10^3 \text{ N}.$$

(c) Let the distance the crate moved relative to the conveyor belt before it stops slipping be $d$, then from Eq. 2-16 ($v^2 = 2ad = 2(f/m)d$) we find

$$\Delta E_{th} = f \cdot d = \frac{1}{2}mv^2 = K.$$

Thus, the total energy that must be supplied by the motor is

$$W = K + \Delta E_{th} = 2K = (2)(216 \text{ J}) = 432 \text{ J}.$$

(d) The energy supplied by the motor is the work $W$ it does on the system, and must be greater than the kinetic energy gained by the crate computed in part (b). This is due to the fact that part of the energy supplied by the motor is being used to compensate for the energy dissipated $\Delta E_{th}$ while it was slipping.
24. From Chapter 4, we know the height $h$ of the skier's jump can be found from

$$v_y^2 = 0 = v_{0y}^2 - 2gh$$

where $v_{0y} = v_0 \sin 28^\circ$ is the upward component of the skier's “launch velocity.” To find $v_0$ we use energy conservation.

(a) The skier starts at rest $y = 20$ m above the point of “launch” so energy conservation leads to

$$mg\Delta y = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy} = 20 \text{ m/s}$$

which becomes the initial speed $v_0$ for the launch. Hence, the above equation relating $h$ to $v_0$ yields

$$h = \frac{(v_0 \sin 28^\circ)^2}{2g} = 4.4 \text{ m}.$$ 

(b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.