



Vectors

MAT208

Sec. 4.1

Fall 2009

Definitions

- A vector in the plane is a 2×1 matrix $\mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ where x and y are real numbers
- Other Notation -- \underline{v} \vec{v} $\langle \quad \rangle$
- The numbers x and y in the definition of a vector are called the components of the vector \mathbf{v} .
- Two vectors $\mathbf{v} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ are equal if $x_1 = x_2$ and $y_1 = y_2$



Geometric Interpretation

- A two-dimensional vector $\mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ has several geometric interpretations
 - 1) A point (x, y) in the plane
 - 2) A directed line segment from the origin to the point (x, y)
 - 3) A directed line segment from the point (x_1, y_1) to the point (x_2, y_2) . Then
$$x = x_2 - x_1, y = y_2 - y_1$$

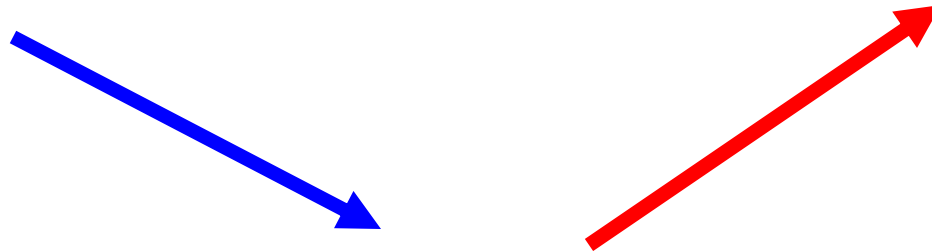
Vectors - Physics vs. Math

■ Physics

- Emphasize physical applications
- Set up problem to simplify analysis

■ Mathematics

- Emphasize geometrical applications
- Analyze the most general case



When we do the same problem, we sometimes use different notation.

Vector Description

- Physics

- Vector is an arrow
 - Direction
 - Magnitude (Amount)

- Vector terms

- Head
- Tail

Applications use all terms interchangeably.

- Mathematics

- Vector is a list
 - Direction
 - Magnitude (Length)

- Vector terms

- Initial point
- Terminal point

Applications use all terms interchangeably.

Vector Units

■ Physics

- All vectors have units
 - Examples

N

m/s

- Typical vector:

$$d = 2.0 \hat{i} + 7.2m \hat{j}$$

■ Mathematics

- Pure vectors don't have units
- In applied examples

N

m/sec

- Typical vector

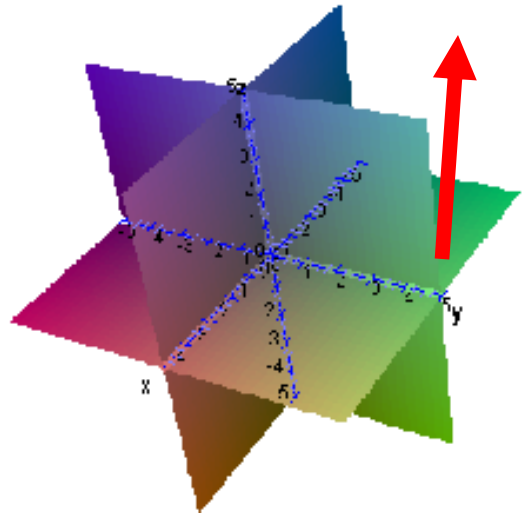
$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$v = \langle 6, -2 \rangle$$

Vector - Physical Quantities

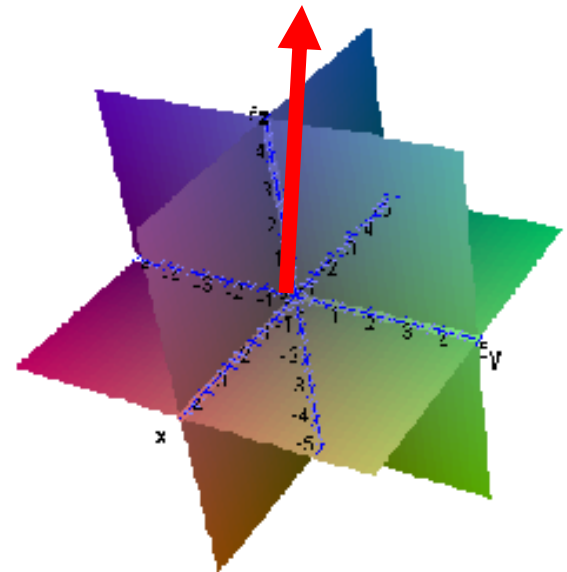
■ Physics

Tail of vector is at point of application



■ Mathematics

Tends to put tail at the origin



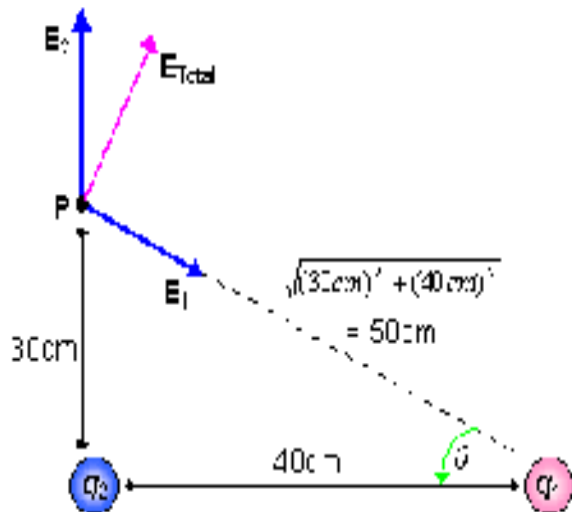
Vector Algebra

- Physics
- Vectors usually given in terms of magnitude and direction.

(Find components)

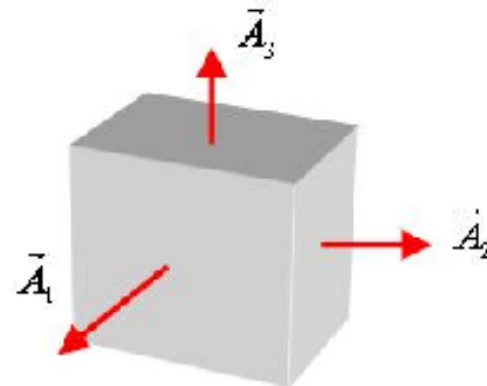
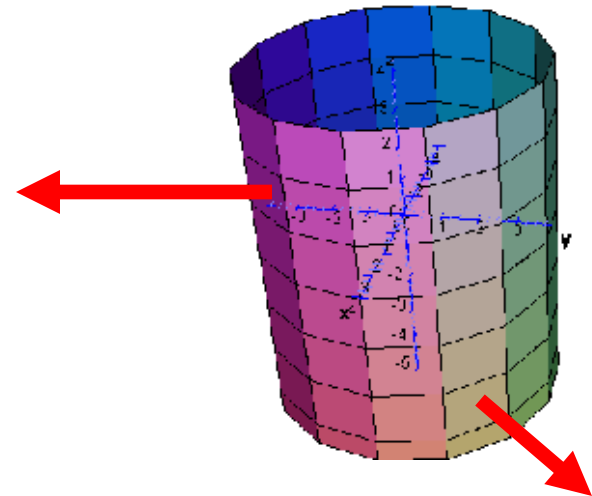
- Mathematics
- Vectors often given in terms of components

$$\vec{v}_1 + \vec{v}_2 = \vec{v}_{1x} + \vec{v}_{2x} \ i + \vec{v}_{1y} + \vec{v}_{2y} \ j$$



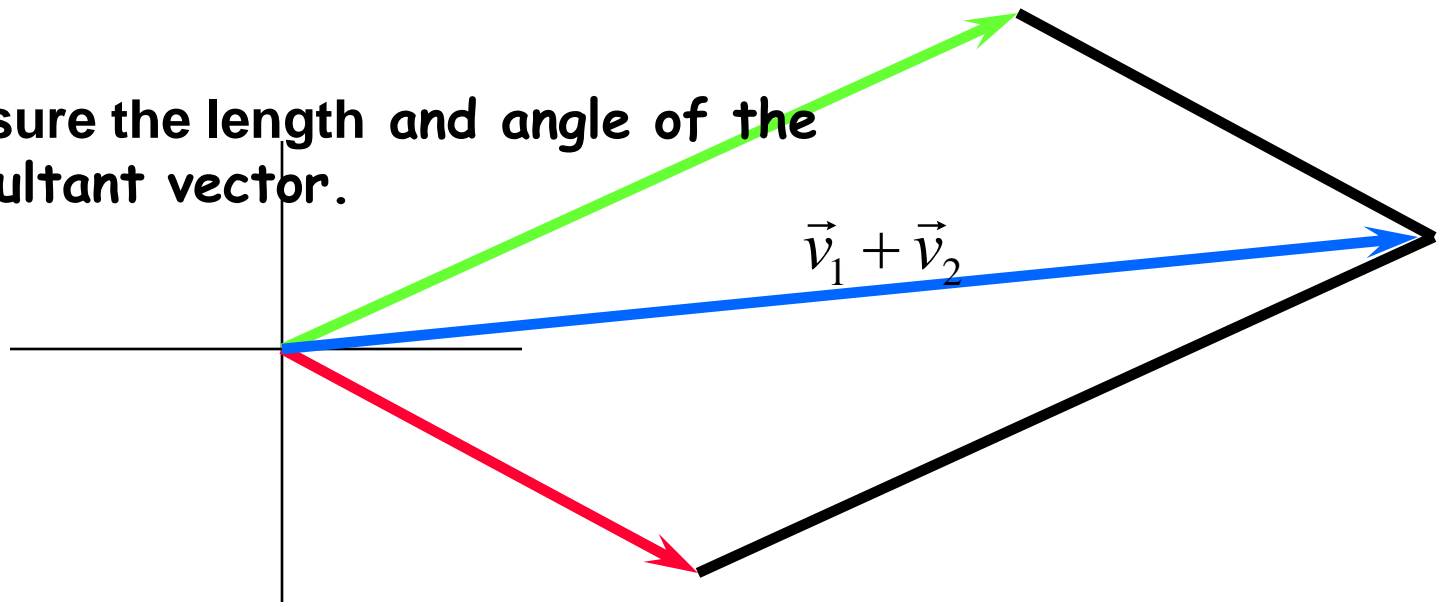
Area Vector Examples

- An area may be represented by a vector
- Vector direction is perpendicular to the area
- Vector points out on a closed surface
- Vector magnitude corresponds to area size



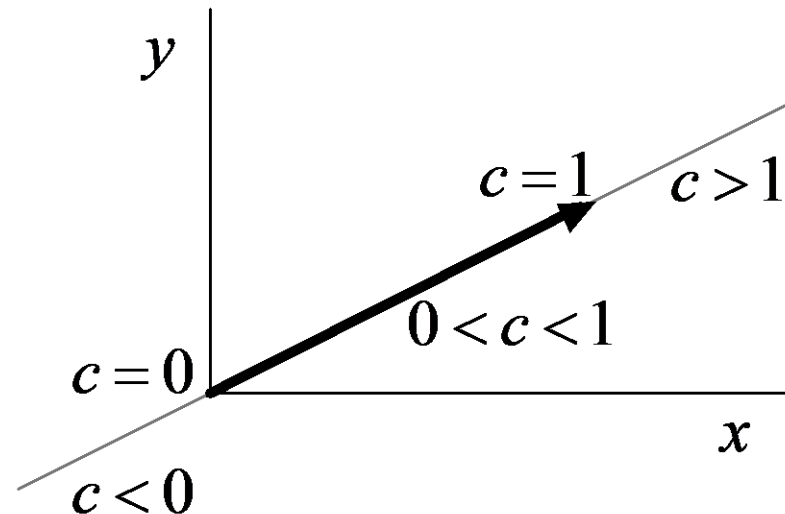
Vector Addition

1. Draw both vectors with proper length and orientation **originating from the same point**.
2. Complete a **parallelogram** using the two vectors as two of the sides.
3. Draw the **resultant vector** as the **diagonal** originating from the tails.
4. Measure the length and angle of the resultant vector.



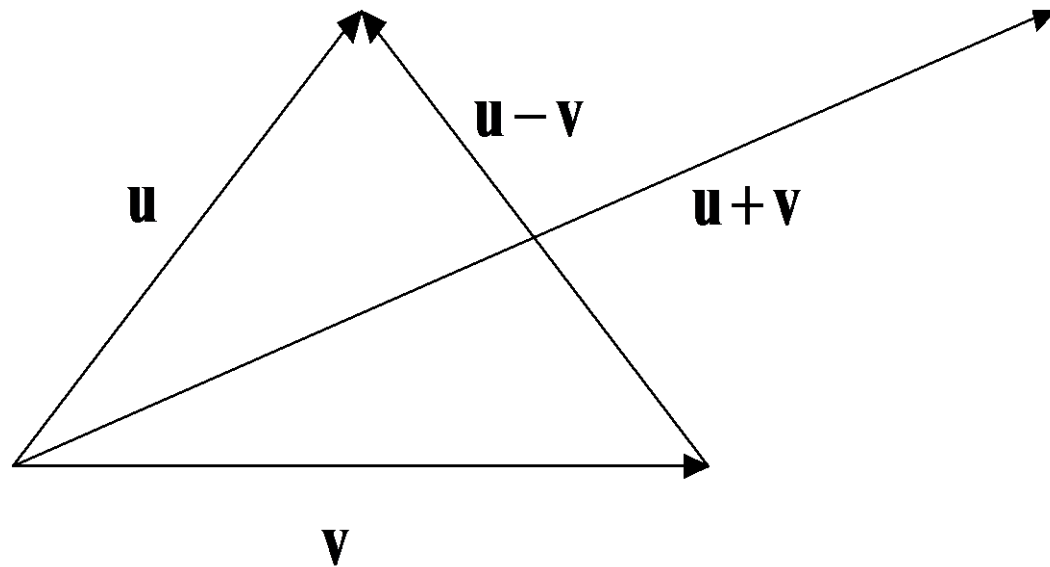
Scalar Multiplication

- Let c be a scalar and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ be a vector.
- The scalar multiple $c \mathbf{u}$ of \mathbf{u} by c is defined as the vector $\begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}$



Vector Subtraction

- Define $\mathbf{u} - \mathbf{v}$ as $\mathbf{u} + (-1)\mathbf{v}$



Basic Properties of Vectors in \mathbb{R}^2 or \mathbb{R}^3

■ Theorem - If \mathbf{u} , \mathbf{v} and \mathbf{w} are vectors in \mathbb{R}^2 or \mathbb{R}^3 and c and d are real scalars, then

a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

b) $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

c) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

e) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

f) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

g) $c(d\mathbf{u}) = (cd)\mathbf{u}$

h) $1\mathbf{u} = \mathbf{u}$

WHY?

- Why can we refer to an ordered pair of real numbers, a 2×1 matrix, and a directed line segment by the same name, “vector”?



Common Properties

- Algebraic behavior
- Features in common
- Abstraction to all objects that have behavior and features in common.

- STAY TUNED!

