



## Liberal Arts Division

### COURSE OUTLINE

MAT208  
Course Number

Linear Algebra  
Course Title

2 per Week  
Class Meetings

4  
Credits

#### **Required Materials**

Introduction to Linear Algebra, Gilbert Strang, 3<sup>rd</sup> Edition, Wellesley-Cambridge Press  
*MATLAB Software*

#### **Catalog Description**

This course is designed as a sophomore level course for majors in Computer Science, Engineering, Physics, Biology, Chemistry or Mathematics and gives an elementary introduction to linear algebra. Systems of linear equations are studied from a computational as well as a theoretical point of view. Topics include the Geometry of  $\mathbb{R}^n$ , Linear Equations and Matrices, Determinants, Independence and Basis, Vector Spaces and Subspaces, the Four Fundamental Subspaces, Orthogonality, Linear Transformations, and Eigenvalues and Eigenvectors. Applications to engineering, statistics, economics, science and other areas will be included and *MATLAB* will be used to gain additional insights into the concepts of linear algebra.

**Prerequisites:** Calculus I

**Corequisites:** None

**Last Revised:** January, 2007

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**Course Goals**

Students will demonstrate through quizzes, examinations, homework and projects the ability to:

1. Generalize the properties of vectors to n-space.
2. Solve  $Ax=b$  for matrices by the method of elimination involving row-reduction, pivots, back-substitution, the invertibility of A and matrix factorization.
3. Define a vector space and subspace.
4. Use the properties of determinants in applications involving the inverse of a matrix and volume problems.
5. Understand the concepts of basis, dimension and linear independence and determine bases for the four fundamental subspaces.
6. Use projections for least-square solutions.
7. Orthogonalize a matrix by Gram-Schmidt factorization.
8. Utilize the algebra of linear transformations.
9. Use the Singular Value Decomposition to diagonalize square and rectangular matrices.
10. Calculate and use eigenvalues and eigenvectors in diagonalization of a matrix and in computing powers of a matrix A
11. Complete projects involving practical applications of linear algebra.

The main learning goals of this Linear Algebra course may be summarized by saying that this course should give students a sound foundation in Linear Algebra that will serve them well in future courses, continue the development of the mathematical maturity of the students, and introduce them to the use of technology tools to tackle more difficult but applications-oriented problems.

**Evaluation of Student Learning**

Students will receive regular feedback on their work through assignments, examinations, lab work, and projects. The syllabus for this course should describe the schedule for these assessment tools and how they will be used to calculate grades. Learning activities will consist of a combination of lectures, lab work and computer assignments. The specific choices for assessment will rest with the instructor. Outside of class, students are expected to do a significant amount of work to achieve learning goals for this course. A typical grading scheme for this course follows:

Midterm Exam	20%	Computer Labs	15%
Project	15%	Final Exam	30%
Graded Assignments	20%		

**Units of Study in Detail****1. LINEAR EQUATIONS AND MATRICES (2 weeks)**

Introduction to Vectors and Linear Combinations  
Systems of Linear Equations  
Matrices and Row Reduction of Linear Systems  
Operations on Matrices  
Matrix Equations and Inverses  
Theory of Linear Systems  
Matrix Factorization

At the end of Unit 1, the student should be able to:

- Define a vector and a linear combination;
- Explain the rules for operations on matrices;
- Use the methods of elimination and factorization on matrices;
- Recognize the connection between the elimination process and factoring a matrix;
- Set up matrix operations using proper technology; and
- Support the validity of results obtained through calculations and through technology.

**2. VECTOR SPACES (4 weeks)**

Vector Space Properties  
Subspaces  
Spanning Set for Subspace of  $\mathbb{R}^n$   
Independence, Basis and Dimension  
Dimensions of the Four Subspaces  
Orthogonality of the Four Subspaces  
Projections  
Least Squares Approximations  
Constructing Orthogonal Bases and Gram-Schmidt

At the end of Unit 2, the student should be able to:

- State the properties of subspaces and the relationships among the four fundamental subspaces;
- Discuss how linear independence, spanning sets, basis and dimension are related;
- Apply least squares approximations to problems to minimize errors;
- Calculate a matrix factorization using the Gram-Schmidt process;
- Estimate the minimum error for a problem using the processes of least squares and proper technology; and
- Compare the methods of matrix factorization studied so far.

### 3. EIGENVALUES and EIGENVECTORS (4 weeks)

Properties of Determinants  
 Cramer's Rule and Inverses  
 Eigenvalues and Characteristic Polynomials  
 Eigenvectors and Eigenspace  
 Diagonalizing a Matrix  
 Positive Definite Matrices  
 Applications to Differential Equations  
 Singular Value Decomposition

At the end of Unit 3, the student should be able to:

- Define determinant, eigenvalue and eigenvector;
- Describe how eigenvalues and eigenvectors are used to find solutions to systems of differential equations;
- Use Cramer's Rule to solve systems of equations and volume problems;
- Analyze the relationship among determinants and number of eigenvalues, determinants and the product of eigenvalues, and the trace and the sum of eigenvalues;
- Summarize the use of the singular value decomposition in applications such as Web search engines and image processing; and
- Conclude, through the use of technology, that the singular value decomposition is necessary for factoring non-square matrices.

### 4. GENERALIZED VECTOR SPACES AND LINEAR TRANSFORMATIONS (3 weeks)

Linear Transformation with its Properties  
 Operations with Linear Transformations  
 Matrix Representations for Linear Transformations  
 Change of Basis  
 Diagonalization and the Pseudoinverse

At the end of Unit 4, the student should be able to:

- Define pseudoinverse for a non-square matrix;
- Identify a linear transformation and find and use its matrix representation;
- Illustrate the process of change-of-basis by building on previous work and definitions;
- Examine the geometry of linear transformations;
- Use technology to construct an example of matrix diagonalization and pseudoinverses; and
- Explain what a wavelet is and how change-of-basis is used in forming them.

**5. APPLICATIONS****(2 weeks)**

Graphs and Networks  
Markov Matrices  
Economic Models  
Cryptography  
Linear Programming  
Fourier Series  
Numerical Linear Algebra  
Statistics

At the end of Unit 5, the student should be able to:

- Name relevant applications for methods studied;
- Explain what routines may be used to solve a given problem;
- Demonstrate an ability to select appropriate calculations for projects;
- Interpret the validity of results;
- Integrate theory, skills and technology to solve a given problem; and
- Compare methods that may be used for a project and choose the one that is most efficient.

Websites for reference:

<http://www.purplemath.com/>

<http://math.about.com/cs/linearalgebra/>

<ftp://joshua.smcvt.edu/pub/hefferon/book/ch1jh.pdf>

<http://www.sosmath.com/matrix/matrix.html>

<http://mathworld.wolfram.com/>

<http://www.ping.be/~ping1339/determ.htm>

<http://n.ethz.ch/student/haertled/eigenexplorer/EigenExplorer.html>

<http://www.numbertheory.org/book/>

<http://www.ping.be/~ping1339/lintf.htm>

<http://www.7stones.com/Homepage/Publisher/linAlgMenu.html>

**Learning Center Resources:** tutors should be available in the learning center. The instructor may wish to hold one office hour in the learning center if no tutors are available.