

Math Anxiety: Personal, Educational, and Cognitive Consequences

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Abstract

Highly math-anxious individuals are characterized by a strong tendency to avoid math, which ultimately undercuts their math competence and forecloses important career paths. But timed, on-line tests reveal math-anxiety effects on whole-number arithmetic problems (e.g., $46 + 27$), whereas achievement tests show no competence differences. Math anxiety disrupts cognitive processing by compromising ongoing activity in working memory. Although the causes of math anxiety are undetermined, some teaching styles are implicated as risk factors. We need research on the origins of math anxiety and on its "signature" in brain activity, to examine both its emotional and its cognitive components.

Keywords

anxiety; mental arithmetic; math competence; working memory; problem solving

My graduate assistant recently told me about a participant he had tested in the lab. She exhibited increasing discomfort and nervousness as the testing session progressed, eventually becoming so distraught that she burst into tears. My assistant remarked that many of our participants show some unease or apprehension during testing—trembling hands, nervous

laughter, and so forth. Many ask, defensively, if their performance says anything about their overall intelligence. These occasionally extreme emotional reactions are not triggered by deliberately provocative procedures—there are no personally sensitive questions or intentional manipulations of stress. Instead, we merely ask college adults to solve elementary-school arithmetic problems, such as $46 + 18 = ?$ and $34 - 19 = ?$

The reactions are obvious symptoms of anxiety, in this case math anxiety induced by ordinary arithmetic problems presented in timed tasks. On the one hand, it is almost unbelievable that tests on such fundamental topics can be so upsetting; knowing that $15 - 8 = 7$ ought to be as basic as knowing how to spell "cat." On the other hand, U.S. culture abounds with attitudes that foster math anxiety: Math is thought to be inherently difficult (as Barbie dolls used to say, "Math class is hard"), aptitude is considered far more important than effort (Geary, 1994, chap. 7), and being good at math is considered relatively unimportant, or even optional.

In this article, I discuss what has been learned about math anxiety across the past 30 years or so, and suggest some pressing issues to be pursued in this area. An important backdrop for this discussion is the fact that modern society is increasingly data and technology oriented, but the formal educational system seems increasingly unsuccessful at educating students to an adequate level of "numeracy," the

mathematical equivalent of literacy (Paulos, 1988).

MATH ANXIETY DEFINED AND MEASURED

Math anxiety is commonly defined as a feeling of tension, apprehension, or fear that interferes with math performance. The first systematic instrument for assessing math anxiety was the Mathematics Anxiety Rating Scale (MARS), published by Richardson and Suinn (1972). In this test, participants rate themselves on the level of anxiety they would feel in various everyday situations, such as trying to refigure a restaurant bill when they think they have been overcharged or taking a math test. My co-workers and I use a shortened version of the test, which yields scores that correlate well with scores obtained using the original test and also has very acceptable test-retest reliability (i.e., an individual who takes the test on different occasions generally receives similar scores). We have also found that for a quick determination, one can merely ask, "On a scale from 1 to 10, how math anxious are you?" Across at least a half-dozen samples, responses to this one question have correlated anywhere from .49 to .85 with scores on the shortened MARS.

There is a rather extensive literature on the personal and educational consequences of math anxiety, summarized thoroughly in Hembree (1990). Perhaps the most pervasive—and unfortunate—tendency is avoidance. Highly math-anxious individuals avoid math. They take fewer elective math courses, both in high school and in college, than people with low math anxiety. And when they take math, they receive lower grades. Highly math-anxious people also espouse negative attitudes toward math, and hold negative self-perceptions

about their math abilities. The correlations between math anxiety and variables such as motivation and self-confidence in math are strongly negative, ranging between $-.47$ and $-.82$. It is therefore no surprise that people with math anxiety tend to avoid college majors and career paths that depend heavily on math or quantitative skills, with obvious and unfortunate consequences.

Interestingly, math anxiety is only weakly related to overall intelligence. Moreover, the small correlation of $-.17$ between math anxiety and intelligence is probably inflated because IQ tests include quantitative items, on which individuals with math anxiety perform more poorly than those without math anxiety. The small correlation ($-.06$) between math anxiety and verbal aptitude supports this interpretation. However, math anxiety is related to several other important characteristics. As conventional wisdom suggests, it is somewhat higher among women than men. The gender difference is rather small, may be particularly apparent in highly selected groups (e.g., college students), and may be partly attributable to a greater willingness on the part of women to disclose personal attitudes. Nonetheless, when we recruited participants for research on math anxiety, we found fewer men than women at high anxiety levels, but just the reverse at low levels (Ashcraft & Faust, 1994).

Individuals who are high in math anxiety also tend to score high on other anxiety tests. The strongest interrelationship is with test anxiety, a $.52$ correlation. Despite the overlap among kinds of anxiety, however, the evidence is convincing that math anxiety is a separate phenomenon. For instance, intercorrelations between alternative assessments of math anxiety range from $.50$ to $.70$, but intercorrelations of math anxiety

with other forms of anxiety range from $.30$ to $.50$. In a particularly clear display of the specificity of math anxiety, Faust (1992) found physiological evidence of increasing reactivity (e.g., changes in heart rate) when a highly math-anxious group performed math tasks of increasing difficulty. When the same participants performed an increasingly difficult verbal task, there was hardly any increase in their reactivity (e.g., Ashcraft, 1995, Fig. 6), and participants with low math anxiety showed virtually no increase in either task.

MATH ANXIETY AND MATH COMPETENCE

An obvious but unfortunate consequence of the avoidance tendency is that compared with people who do not have math anxiety, highly math-anxious individuals end up with lower math competence and achievement. They are exposed to less math in school and apparently learn less of what they are exposed to; as a result, they show lower achievement as measured by standardized tests (e.g., Fennema, 1989). The empirical relationship is of moderate strength (a correlation of $-.31$ for college students), but sufficient to pose a dilemma for empirical work. That is, when highly math-anxious individuals perform poorly on a test, their poor performance could in fact be due to low competence and achievement rather than heightened math anxiety. If the relationship between anxiety and competence holds for all levels of math difficulty, then variations in competence will contaminate any attempt to examine math performance at different levels of math anxiety.

Fortunately, there are ways out of this dilemma. One is to test additional samples of participants on

untimed, pencil-and-paper versions of the math problems studied in the lab. For example, we (Faust, Ashcraft, & Fleck, 1996) found no anxiety effects on whole-number arithmetic problems when participants were tested using a pencil-and-paper format. But when participants were tested on-line (i.e., when they were timed as they solved the problems mentally under time pressure in the lab), there were substantial anxiety effects on the same problems.

We have also taken a second approach (see Ashcraft, Kirk, & Hopko, 1998). In brief, we administered a standard math achievement test to individuals with low, medium, or high math anxiety, and replicated the overall result reported by Hembree (1990; i.e., math achievement scores decrease as math anxiety increases). But we then scored the achievement test to take advantage of the line-by-line increases in difficulty. With this scoring method, we found that there were no math-anxiety effects whatsoever on the first half of the test, which measured performance on whole-number arithmetic problems. Anxiety effects were apparent only on the second half of the test, which introduced mixed fractions (e.g., $10\frac{1}{4} - 7\frac{2}{3}$), percentages, equations with unknowns, and factoring. For these problems, there was a strong negative relationship between accuracy and math anxiety. Thus, individuals with high levels of math anxiety do not have a global deficit in math competence, and they can perform as well as their peers on whole-number arithmetic problems. Investigations of higher-level arithmetic and math, though, do need to take the competence-anxiety relationship into account.

There is still reason to be somewhat suspicious of this relationship between anxiety and competence, however. Effective treatments for math anxiety (see Hembree, 1990,

Table 8) have resulted in a significant improvement in students' math achievement scores, bringing them nearly to the level shown by students with low math anxiety. Because the treatments did not involve teaching or practicing math, the improvement could not be due to a genuine increase in math competence. We suspect instead that these students' original (i.e., pre-treatment) math competence scores were artificially low, depressed by their math anxiety. When the anxiety was relieved, a truer picture of their competence emerged.

COGNITIVE CONSEQUENCES OF MATH ANXIETY

Our original studies were apparently the first to investigate whether math anxiety has a measurable, on-line effect on cognitive processing, that is, whether it actually influences mental processing during problem solving. In our early studies (Ashcraft & Faust, 1994; Faust et al., 1996), we found that math anxiety has only minimal effects on performance with single-digit addition and multiplication problems. One anxiety effect we did find, however, was in a decision-making process sensitive to "number sense" (Dehaene, 1997)—when making true/false judgments, highly math-anxious individuals made more errors as the problems became increasingly implausible (e.g., $9 + 7 = 39$), whereas low-anxiety participants made fewer errors on such problems.

Arithmetic problems with larger numbers (e.g., two-column addition or multiplication problems), however, showed two substantial math-anxiety effects. First, participants at high levels of anxiety routinely responded rapidly to these problems, sometimes as rapidly as participants with low anxiety, but

only by sacrificing considerable accuracy. This behavior resembles the global avoidance tendency characteristic of highly math-anxious individuals, but at an immediate, local level: By speeding through problems, highly anxious individuals minimized their time and involvement in the lab task, much as they probably did in math class. Such avoidance came at a price, however—a sharp increase in errors.

Second, the results showed that addition problems with carrying were especially difficult for highly math-anxious individuals. In particular, the time disadvantage for carry versus no-carry problems was three times larger for participants with high anxiety (753 ms) than for those with low anxiety (253 ms), even aside from the difference in accuracy between the two groups. Our interpretation was that carrying, or any procedural aspect of arithmetic, might place a heavy demand on working memory, the system for conscious, effortful mental processing. In other words, we proposed that the effects of math anxiety are tied to those cognitive operations that rely on the resources of working memory.

In an investigation of this possibility, Kirk and I (Ashcraft & Kirk, 2001) tested one- and two-column addition problems, half requiring a carry. We embedded this test within a dual-task procedure, asking our participants to do mental math, the primary task, while simultaneously remembering random letters, a secondary task that taxes working memory. Two or six letters were presented before each addition problem, and after participants gave the answer to the problem, they were asked to recall the letters in order. We reasoned that as the secondary task became more difficult (i.e., when more letters had to be held in working memory), performance on the primary task might begin to degrade, in either speed or accuracy. If that hap-

pened, we could infer that the primary task indeed depended on working memory, and that the combination of tasks began to exceed the limited capacity of working memory.

When the addition problem involved carrying, errors increased substantially more for participants with high math anxiety than for those with low anxiety (Ashcraft & Kirk, 2001, Experiment 2). Moreover, as we predicted, this was especially the case when the secondary task became more difficult, that is, with a six-letter memory load. On carry problems (e.g., $6 + 9$, $27 + 15$), highly anxious individuals made 40% errors in the heavy-load condition, compared with only 20% errors for individuals with low anxiety in the high-load condition and 12% errors for both groups in the light-load condition. In the control conditions, with each task performed separately, the comparable error rates were only 16% and 8%. These results could not be attributed to overall differences in working memory. That is, we examined the participants' working memory spans (the amount of information they were able to remember for a brief amount of time) and found no differences between the groups when spans were assessed with a verbal task. But span scores did vary with math anxiety when they were assessed with an arithmetic-based task.

These results are consistent with Eysenck and Calvo's (1992) model of general anxiety effects, called processing efficiency theory. In this theory, general anxiety is hypothesized to disrupt ongoing working memory processes because anxious individuals devote attention to their intrusive thoughts and worries, rather than the task at hand. In the case of math anxiety, such thoughts probably involve preoccupation with one's dislike or fear of math, one's low self-confidence, and the like. Math anxiety lowers

math performance because paying attention to these intrusive thoughts acts like a secondary task, distracting attention from the math task. It follows that cognitive performance is disrupted to the degree that the math task depends on working memory.

In our view, routine arithmetic processes like retrieval of simple facts require little in the way of working memory processing, and therefore show only minimal effects of math anxiety. But problems involving carrying, borrowing, and keeping track in a sequence of operations (e.g., long division) do rely on working memory, and so should show considerable math-anxiety effects. Higher-level math (e.g., algebra) probably relies even more heavily on working memory, so may show a far greater impact of math anxiety; note how difficult it will be, when investigating high-level math topics, to distinguish clearly between the effects of high math anxiety and low math competence.

GAPS IN THE EVIDENCE

Math anxiety is a bona fide anxiety reaction, a phobia (Faust, 1992), with both immediate cognitive and long-term educational implications. Unfortunately, there has been no thorough empirical work on the origins or causes of math anxiety, although there are some strong hints. For instance, Turner et al. (2002) documented the patterns of student avoidance (e.g., not being involved or seeking help) that result from teachers who convey a high demand for correctness but provide little cognitive or motivational support during lessons (e.g., the teacher "typically did not respond to mistakes and misunderstandings with explanations," p. 101; "he often showed annoyance when students gave wrong an-

swers He held them responsible for their lack of understanding," p. 102). Turner et al. speculated that students with such teachers may feel "vulnerable to public displays of incompetence" (p. 101), a hypothesis consistent with our participants' anecdotal reports that public embarrassment in math class contributed to their math anxiety. Thus, it is entirely plausible, but as yet undocumented, that such classroom methods are risk factors for math anxiety.

Other gaps in the evidence involve the cognitive consequences of math anxiety, including those that interfere with an accurate assessment of math achievement and competence. My co-workers and I have shown that the transient, on-line math-anxiety reaction compromises the activities of working memory, and hence should disrupt performance on any math task that relies on working memory. The mechanisms for this interference are not yet clear, however. It may be that intrusive thoughts and worry *per se* are not the problem, but instead that math-anxious individuals fail to inhibit their attention to those distractions (Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998).

Finally, as research on mathematical cognition turns increasingly toward the methods of cognitive neuroscience, it will be interesting to see what "signature" math anxiety has in brain activity. The neural activity that characterizes math anxiety should bear strong similarities to the activity associated with other negative affective or phobic states. And our work suggests that the effects of math anxiety should also be evident in neural pathways and regions known to reflect working memory activity.

Recommended Reading

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Note

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