COURSE OUTLINE

Course Number	Course Title	Credits
MAT 251	**Calculus III**	**4**
Hours: Lecture/Lab/Other	**Co- or Pre-requisite**	**Implementation Semester & Year**
4 lecture	MAT152 with a minimum C grade or better, successful completion of an equivalent course, or approval of the department chair.	**Fall 2022**

Catalog description:
Calculus III, Multivariable Calculus, applies the techniques and theory of differentiation and integration to vector-valued functions and functions of more than one variable. Topics include vectors, vector-valued functions, quadratic surfaces in three-space, spherical coordinate systems, partial derivatives and multiple integrals, Lagrange multipliers, the theorems of Gauss, Green and Stokes, and line integrals.

General Education Category:
Goal 2: Mathematics

Course coordinator:
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Required texts & Other materials:
Calculus: Volume 3, Edwin Herman and Gilbert Strang
https://openstax.org/details/books/calculus-volume-3

Course Student Learning Outcomes (SLO):
Upon successful completion of this course the student will be able to:

1. Define, calculate, and apply the concepts of vectors, dot, and cross products. [Supports ILG #2, 11, PLO #1 - 4]
2. Calculate arc length, unit tangent and normal vectors, curvature, and surface area. [Supports ILG #2, 11, PLO #1 - 4]
3. Describe the basic analytic geometry of lines, planes and spheres in three space, both with vector equations and with scalar equations. [Supports ILG #2, 11, PLO #1 - 4]
4. Perform computations with vector-valued functions. [Supports ILG #2, 11, PLO #1 - 4]
5. Analyze and evaluate limits and continuity of multivariable functions. [Supports ILG #2, 11, PLO #1 - 4]
6. Construct representations of functions of two variables as surfaces and level curves. [Supports ILG #2, 11, PLO #1 - 4]
7. Calculate and apply partial derivatives and gradient fields. [Supports ILG #2, 11, PLO #1 - 4]
8. Set up and solve optimization problems for functions of several variables and find extreme values by a variety of methods, including Lagrange multipliers. [Supports ILG #2, 11, PLO #1 - 4]
9. Apply the properties of curves, surfaces and vector fields to velocity, curl, and divergence problems. [Supports ILG #2, 11, PLO #1 - 4]
10. Calculate double and triple integrals, establish the relationship of the integrals with geometric regions of integration in both rectangular, polar, and spherical coordinate systems, change the order of integration for double and triple integrals, and apply them to area and volume problems. [Supports ILG #2, 11, PLO #1 - 4]

11. Calculate line and surface integrals by applying the fundamental theorems of calculus. [Supports ILG #2, 11, PLO #1 - 4]
12. Determine path independence and state and use Green's, Gauss's, and Stokes's theorems. [Supports ILG #2, 11, PLO #1 - 4]

Course-specific Institutional Learning Goals (ILG):

Institutional Learning Goal 2. Mathematics. Students will use appropriate mathematical and statistical concepts and operations to interpret data and to solve problems.
Institutional Learning Goal 11. Critical Thinking: Students will use critical thinking skills understand, analyze, or apply information or solve problems.

Program Learning Outcomes for Mathematics AS (PLO)

1. Apply a range of mathematical skills spanning fundamental concepts to more advanced mathematical concepts.
2. Apply quantitative knowledge, including the required technological skills and theoretical knowledge.
3. Demonstrate critical thinking skills to solve real world problems using mathematical modeling.
4. Communicate methods of solutions and results to problems using mathematical language and notation.

Units of study in detail – Unit Student Learning Outcomes:

Unit I Vectors [Supports Course SLOs #1, 2, 3]

Learning Objectives

The student will be able to:

- Define, understand and use vectors in Euclidean 2- and 3-space.
- Compute dot products and cross products, and interpret them as work, area, volume.
- Resolve a vector into its components given a basis of unit orthogonal unit vectors
- Find the normal vector given the equation of a plane and find the equation of a plane given a normal and a point; similarly in two dimensions for a line.
- Write equations for and identify and sketch quadric surfaces.
- Convert to and from cylindrical and spherical coordinates.
- Apply cylindrical coordinates to problems involving symmetry around an axis.
- Apply spherical coordinates to problems involving symmetry about a point.

Unit II Vector - Valued Functions [Supports Course SLO #4]

Learning Objectives

The student will be able to:

- Interpret, apply and visualize functions of two (or more) variables.
- Calculate derivatives and integrals of vector-valued functions.
- Parameterize a curve with respect to arc length and find the arc length.
- Define curvature as a measure of how quickly the curve changes direction.
- Calculate curvature using derivatives and cross products.
- Define, calculate, and use the orthogonal vectors forming the TNB (unit tangent, unit normal, and unit binormal) frame.
- Represent motion as parameterized curves in space and be able to compute velocity and acceleration vectors.
- Demonstrate knowledge of Kepler's laws of motion and their historical significance to mathematics.

Unit III **Partial Derivatives [Supports Course SLOs #5, 6, 7, 8, 9]**

Learning Objectives

The student will be able to:

- Define what it means for a function of several variables to be continuous and decide when such functions are continuous.
- Define, interpret, and compute partial derivatives and understand Clairaut's Theorem.
- Approximate a function of two variables by using the tangent plane to create a tangent plane approximation or local linear approximation to the function.
- Establish and use the chain rule for taking the derivative of a multivariable function.
- Compute, interpret, and apply directional derivatives and gradient vectors.
- Understand the significance of the gradient and apply it as the direction of greatest increase or as the normal to level curves and surfaces.
- Apply the second derivative test and extreme value theorem for find maximum and minimum values for functions of two variables.
- Apply the method of Lagrange multipliers to find maximum and minimum values of multivariable functions.

Unit IV **Multiple Integrals [Supports Course SLO #10]**

Learning Objectives

The student will be able to:

- Integrate functions of several variables over regions in the plane or in space.
- Use multiple integration to find surface area and volumes.
- Change the order of integration for double and triple integrals and understand Fubini's Theorem.
- Integrate using polar, spherical, or cylindrical coordinates.
- Integrate by change of variables and by using the Jacobian of the transformation.
- Explore physical applications of multiple integrals including computing electric charge, center of mass, moment of inertia, probability, and expected value.

Unit V **Vector Calculus [Supports Course SLOs #11, 12]**

Learning Objectives

The student will be able to:

- Sketch a vector field given a formula.
- Recognize the characteristics of a conservative vector field.
- Compute line integrals over a curve.
- Define path independence and demonstrate that line integrals of conservative fields are independent of path.
- State the equivalence of path independent vector fields and gradient vector fields.
- State and apply Green's Theorem in the plane.
- Define the operations of curl and divergence that can be performed on vector fields.
- Calculate, interpret, and apply the curl of a vector field.
- Calculate, interpret, and apply the divergence of a vector field.
- Use the curl and divergence to rewrite Green's theorem.
- Set up and use surface integrals for various applications.
- State and apply Stokes's theorem.
- Interpret, apply, and provide a justification for the divergence theorem, also called Gauss's theorem.

Evaluation of student learning:

Tests, quizzes, homework assignments and projects may be used in evaluating the students' progress throughout the course depending on the individual instructor. It is suggested that four unit tests and a final

exam be used in evaluating the students' progress. A suggested day-by-day schedule and suggested homework problems should be available to the students. A final exam must be given in the course.

A possible plan for determining the students' final grades is as follows:

Four Unit Tests	60%
Cumulative Final Exam	25%
Homework, Projects and Quizzes	15%